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RIVER HYDRAULICS.

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PRESENTED DECEMBER 6TH, 1899.

WITH DISCUSSION.

The following is a brief outline of the methods and results of a systematic study of data taken on the Mississippi River and its tributaries. The system of rivers, the principal drainage divisions and a number of locations to which it will be necessary to refer, are shown on the map (Fig. 1).

As the writer is satisfied that this study has now reached such a point that it will enable river engineers to see clearly what observations should be made and what conclusions might be drawn from them, thereby being the means of saving much time and money, he has ventured to give it as it is, notwithstanding the fact that it contains a number of questions as yet unanswered.

As an introduction, it may be noted that, in working in an established field of science, the relation between phenomena and their fundamental principles is all that need be considered, but it is somewhat different where the phenomena are simply a mass of facts and no fundamental principles are given. In such cases the first step is to find the relations between the given facts; the fundamental principles in which these relations are included will come later.

This is the manner in which this study was made, and its results here presented. The data are subdivided into the different classes in which these simple relations of fact were first sought, and an analysis is then given in which these again are all included in a fundamental relation.

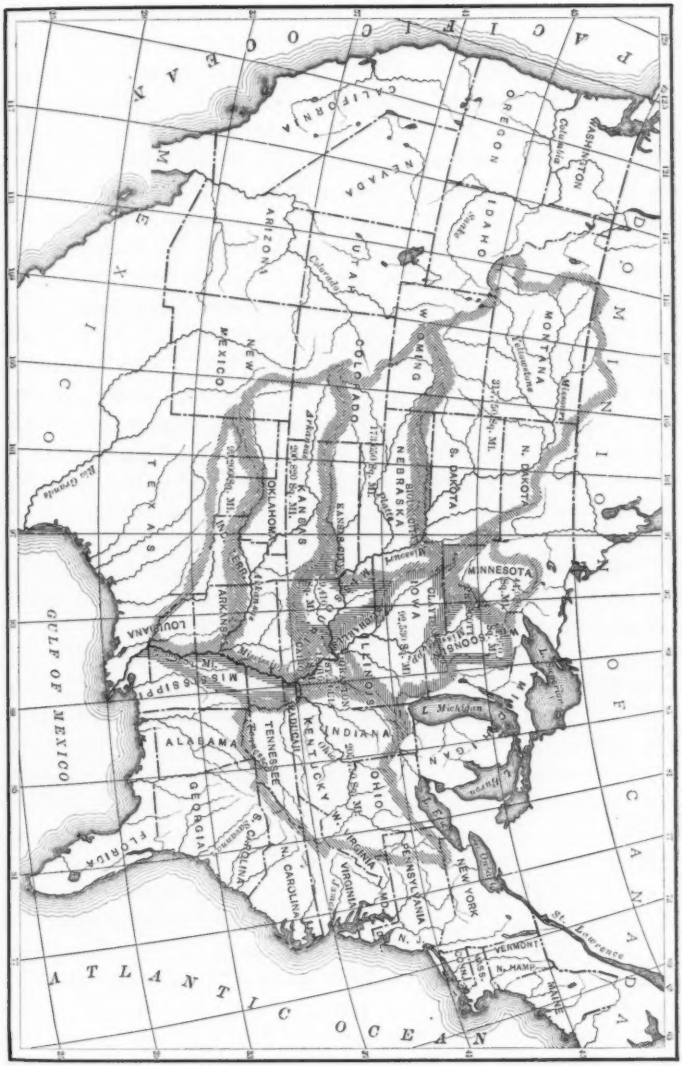
In presenting these relations of fact, each of which has been dug slowly out of great masses of data on various rivers, in some cases covering periods of half a century, the writer has in each case used as an illustration a single instance of the relation in its simplest form, and has noted briefly the variations from this simple form which may occur, and their causes. Thus, the relation between the gauge readings along the river for its own floods is one of these primary relations, while the effects of a flood out of a tributary between any two of the gauges, or the cutting through a bend and the total change of channel near one of them, are considered as variations.

Anyone may test the relation easily, by referring to the published data in the cases given; but, after that, it should be understood that the relation is not to be discredited by selecting other periods, in which it is not so simple, without at the same time considering their variations. The whole conclusion lies often in hundreds and sometimes even in thousands of plates of data, which, of course, cannot be given here, and, so far, the writer must be trusted. The test comes in the application of the final analysis, and, if this bears the test, it will not be necessary for anyone else to go through again the laborious process which he has had to follow.

Slope and Section Data.

It is not the purpose here to attempt to add anything to the data or the theory of velocity-slope relations; nor, indeed, important as they are to the engineer who lays a pipe or builds a conduit, to consider them even directly. Where man makes the form and sets the slope, the resulting flow is the one thing that he wants to know, but where the flow makes its own form, and that a very irregular one, and so distributes its slope that it may be almost all concentrated at certain points at one stage, with little or none there at a different stage, there is not much use in trying to express it as $v = c\sqrt{r\,s}$, for r and s are even more difficult to measure than v.

Of course the fall is determined quite accurately over long reaches,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GEORGIA
FLORIDA
S. CAROLINA
N. CAROLINA
VIRGINIA
W. VIRGINIA
PENNSYLVANIA
NEW YORK
VERMONT
N. HAMP.
MAINE
GULF OF MEXICO
ATLANTIC OCEAN
DOMINION OF CANADA

FIG. 1.

but to determine the values of the mean velocity and the mean hydraulic radius which correspond to it may require a year's observations and survey, while if these are taken in the selected section the slope there may be found to be even reversed, or the water surface may be found to slope in the opposite direction to the flow. It is not at all impossible that, taking a hundred miles of river a mile wide, this formula may in certain cases express the general relations quite as well as it does in 100 ft. of pipe 1 ft. in diameter, but such a relation in the river is wholly useless; while it is equally possible that, if the relation were sought in a 1-ft. length of the pipe, it would be found altogether as erratic as it is for the selected section of the river.

True, there is a difference between the foot of pipe and the mile of river; the theoretical equivalent is probably given more nearly by a relation between resistance and momentum, or the lengths of a fall equal to the given velocities; but, in addition to this, the give and take of stored energy in the river is to be considered. The Lower Mississippi at its bank-full stage has at least a range of from 10 000 to 30 000 foot-tons in the energy of its flow, and in the play of such a balance wheel local slope-velocity relations very naturally are obliterated.

Not only, however, are no such local relations to be looked for in rivers, but also, even if such a general relation should exist at one stage, such a general relation at all stages by no means follows. In many cases the alternations of pool and bar at low water do not differ greatly from a chain of small reservoirs, each emptying through a weir into that below it; and in that case the level and the discharge of each are not affected in the least by the vertical distance which one may be put below the other. True, they may wear under the fall, unless the pool is deep enough and the material is stable enough, but the only slope relation there is this limiting relation with the size of the pools and the character of the bottom.

Where the bed is unyielding and such a condition has simply been formed in the slow process of geological periods, the discharge from pool to pool is no function of slope whatever, but a function of stage. Were the connections plain weirs of given forms, it might be calculated for any level, but as it is more in the nature of a chain of drowned weirs of very irregular forms, the only way to get it is to measure it. In either case, however, it is wholly independent of the different slopes

between the rising and falling river, and it is determined solely by the pool level.

But the concentration of slope between the pools at low water may be entirely reversed at high water, and the narrow sections there control the flow into the series of shallower but broader levels between them; or in this case, the type of control has passed from that of a horizontal weir to that of a vertical orifice, but one is just as arbitrary and just as independent of general slope as the other.

However, in passing from one of these extremes to the other, the river would pass through a stage where something like a general slope relation might be said to control the flow; and here it would probably have a larger discharge on the rising river and a less discharge on the falling river, each in some proportion to the rapidity of the rise or fall. And this is the only point in such a river where velocity-slope relations, even if they could be found, would have any application whatever to its phenomena.

It may be well, finally, to note distinctly that the physics of such a river may be thoroughly known and fully understood, but this would hardly be called hydraulics; for, in the main, it begins and ends in a system of observed facts, and conventional hydraulics throws not a ray of light on the actual proportion in which its elements should stand; they have all simply to be measured.

So much for the river which is not alluvial; what follows will deal strictly with the alluvial river, where the flow handles readily the material of its bed. In such a river, then, that limiting relation referred to, of its slope with its form, is an actual one. In simple cases, this is easily made a subject of experiment. If material from the bed of any such river is put on an inclined plane, a straight channel made through it, and if a given flow which will move the material is turned into it, it will reshape its section until it takes one and only one form for a given inclination.

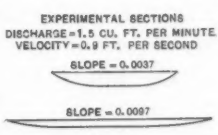

FIG. 2.

Two sections from such a series of experiments made by the writer are given in Fig. 2 to show this effect of slope on a straight channel. They serve very well to illustrate the wide difference in the types of different rivers and its cause, from say, the Platte River at one extreme, where, with a fall of some 6 ft. or more to the mile, it simply runs over the surface of the country, to

the Lower Mississippi at New Orleans at the other, where, with little or no slope, it maintains channel depths 100 to 200 ft. below the Gulf level.

But alluvial rivers are not straight, and neither would these experimental sections remain so. Such an equilibrium is simply a balance of forces, and is not necessarily the only one or the most stable one. Indeed, as the higher slopes are experimented with, the equilibrium of the straight reach is seen very clearly to be less and less stable. It still marks a well-defined point in the experiment at which the flow can be cut off and the section measured, but at the extremes there is really no time when the movement of material in the straight reach can be said to have absolutely stopped everywhere, and it goes on to the formation of bends and crossings, each with more or less accident in its location, with hardly a pause at this, its first and simplest but less stable form of equilibrium.

With a given discharge and slope, the equilibrium of the straight reach has the advantage of being everywhere an identity of form which may be reproduced readily and recognized easily. But though its final equilibrium is not an identity of form anywhere, and its accidental distribution of bends and crossings cannot be reproduced, as a whole, it is no less certainly an identity of effects for the same causes.

But, even with all the freedom in the variations of bend development which the final equilibrium offers, this is but the element of the actions which are going on in the alluvial river. For a given discharge, the bed is everywhere adapting itself to the given slope, while readjusting this slope over pools and bars to its most stable equilibrium, but the river in its season has all discharges from its high-water to its low-water extremes, and different extremes in different seasons; and as these all work each other over, and the results of all are overlaid in the existing conditions at any time, they very naturally produce the endless change of section and the infinite variety of form which are found in the river, the special studies of which are perhaps as fruitless as the study of the differences which may be found in the leaves of the same tree.

To measure this up with anything like thoroughness in a large river, in the first place, requires a great deal of field work, and then begins the difficulty of knowing what to do with it. The general type of the river and the geographical outlines of its bend development

are evident, while the distribution of slope resulting therefrom may be followed roughly through the different forms of the river in its pools and crossings. But, to go further and try to express it in systems of averages, or to follow the general method of compiling it all in the elements of cross-sections, can hardly be said to have been found to be satisfactory, either theoretically or practically. The phenomenon is some form of sequence which no known elements fully express, while to average things which are necessarily different is to obscure their meaning more or less.

However, it is important to note here certain facts in regard to cross-sections. On the Lower Mississippi, where they have been studied most thoroughly, it is plain that the cross-section at any point may not represent the bed of the river there within the limits of some 8 or 10 ft. That is, the mean elevation of the bottom on this line may have such a variation from time to time, while neither the general regimen of the reach nor the surface level of a given stage there have changed perceptibly. Perhaps a special term is needed here to indicate this bed of the river, which, for a given flow, certainly determines the surface level, and it will be called in this sense the "bed in train."

So long as the surface level of a given stage does not change in any reach of river, the bed in train under it certainly remains the same. But when the mean depths of the cross-sections are plotted as elevations of mean bottom on the same profile as this surface, they are generally found to be very irregular; they have sharp ups and downs of some 10 ft. or more, while the corresponding surface changes are measured in tenths or even hundredths of a foot. And, finally, with no change whatever in the regimen of the reach, when one or more of the same sections are re-sounded from time to time, and their mean bottoms show just a like variation of level, there is no room left for the least doubt that the data of sections has simply an accidental divergence from the bed in train of fully this amount.

Indeed, the more thoroughly the bottom of the river is known, the more this accidental variation, which really means nothing, is recognized. At high water sand waves 10 or 15 ft. high may be formed here, while there eddies may be cutting great holes in the bottom; and, with no change in the general conditions of a reach, the bottom at a fixed point in the Lower Mississippi may be found at elevations

differing by as much as 20 to 30 ft., and the mean bottom of a line across it by at least 8 or 10 ft. In smaller rivers these limits are, of course, correspondingly less, but, with their higher slopes, such changes may be even more sudden and erratic.

It is very evident, then, that the bottom anywhere, which at that point represents a real value of the sequence of phenomena in a river

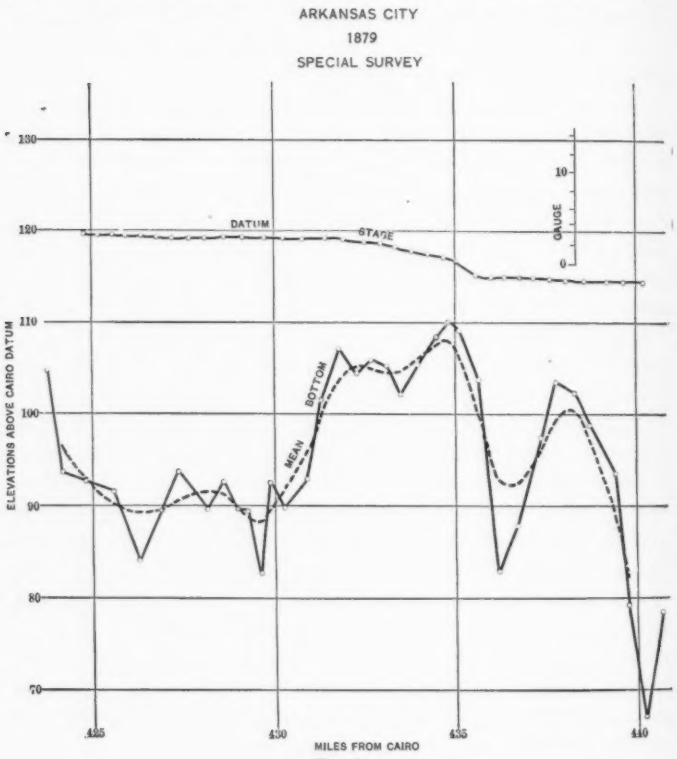

FIG. 3.

is something altogether larger than the bottom on a line there, say half a mile across it, or a like line in any other direction. The local cross-section should doubtless have the most weight, but the bed in train is a part of the condition for some distance, both above and below it.

Some effort has been made to bring the cross-section data together in this form. Taking their elevations of bottom in continuous means of threes, dropping out an upper one and taking in the next lower one at each step, and again repeating the same process on these first means, generally succeeds at last in reducing it to a fairly regular sequence; and it certainly has the merit of bringing the results of a great mass of data together into a very simple form in which they can be seen as a whole and compared with other cases.

The results of a hydrographic survey on the Lower Mississippi in the vicinity of Arkansas City, shown in Fig. 3, will serve to illustrate this. The dashed line of mean bottom is the special sequence deduced by this process of lapped means, while the line joining the plotted points shows the local irregularities in the cross-sections. Each of these, however, in this case, is in general the average of two or more sections often with fifty or more located soundings to each; and, altogether, they have less of this accidental variation of bottom than any of the like data which the writer has yet worked over.

Either of these lines of mean bottom shows the pool and bar formation of this reach very well. It should be noted, however, that probably neither of them shows to its full height the crest of the bar which really controls the level of the upper pool at low water, while in the shorter bars of smaller rivers their failure in this respect would very probably be even greater.

This, however, is about as far as the direct study of form, in an alluvial river, has been carried, and, as before stated, it does not seem that much is to be gotten in this process of trying to understand the river from the bottom up. Practically, every step which the writer has made in river hydraulics, has been made in studies following exactly the opposite process; but before taking up these, in which the whole point of view will be simply reversed, there is a generalization which the hydrographic surveys give, and which perhaps here it may be best to state.

This generalization rests on the observed fact that the average annual caving in of the banks, over hundreds of miles in some of the rivers, is at a rate which would altogether fill them up in a lifetime; and it follows that, for the river to maintain its channel in such a case, as it does, there must be going on annually some equal and opposite process of clearing this out. This simply considers a lateral action,

whatever the movement of material in the direction of flow may be, when a reach, say like that of the Lower Mississippi from White River to Vicksburg, is taken, nearly a mile wide and some 200 miles long, the difference between the material which comes in at the upper end and that which goes out at the lower would certainly have no appreciable effect on such a prism; while the bank erosion there is not far from a fill of 1½ ft. annually over this whole bottom. The clearing out of this quantity annually is, therefore, necessarily, the work simply of a lateral resultant.

How and when it may act will be better understood later. It is enough here to know that its action is a fact. It is not deduced from experiments on slope equilibriums, nor found directly in the forms of cross-sections. But, certainly in that light, much which has been said about alluvial rivers filling themselves up with the matter which they carry in suspension, is, to say the least, trivial. The difficulty which the water-works engineer has in getting the suspended matter out of the river water when once it has gotten into it, does not encourage the idea that any great additional load will be thrown on the lateral resultant from this cause.

While a river in all its upper part may be cutting down its bed, in the slow process of geological periods, undoubtedly in its lower alluvial part it may be building it up by the same process, and, with its alluvial part relatively short, this action may be even appreciable in the course of centuries; but in systems where the alluvial reaches extend up for thousands of miles, it seems hardly worth considering, and, certainly, not to the exclusion of the equally palpable fact that the alluvial river is also keeping its bed down, or otherwise its channel would fill up by the fall of material into it, and this also in periods which, in comparison with the other action, are relatively immediate.

Discharge and Gauge Data.

Having come now to the point at which is recognized the difficulty of getting anything satisfactory out of the cross-sections to express for a given stage the bed in train, it is proposed, in what follows, simply to take a surface measure for it. Laws of flow are there, whether or not they are known, and in these laws the bed determines the surface. Either can be taken as a measure of the other, and the question of how this measure is to be interpreted can be considered later.

In any case, if the flow into and out of a given reach is the same and the surface level is fixed, there is just so much water there all the time, and the bed, as a whole, is equally fixed under it. In the alluvial river this may be only an instantaneous equilibrium, neither the bed nor the surface may remain fixed, and the surface especially may be changing on account of changes in the amount of water flowing. There are also reasons for thinking that it may change on account of changes in the conditions of flow attending the rise or fall of the river, or the entrance of floods from tributaries; but if none of these will account for a change found in the surface, it must be accounted for by a change in the bed of the river under it.

The first thing required, however, to identify this surface measure, is to determine the flow which corresponds to it. If the river is on a stand through a given reach, with discharge everywhere the same, its surface level may be said to measure everywhere the bed under it; but this alone gives nothing which can be brought into comparison with any other part of the river, or indeed with the same river at another time. But when the discharge is also known, and the reach comes back, say after a flood, to the same discharge with a different surface level, this gives an absolute measure of the changes which have taken place in the bed of the river at that stage, though as yet this measure may not be interpreted.

At any location on the river, where a gauge record is kept, the readings give the surface levels from day to day through all the changes of the seasons, but to use these for the purposes in hand, or as given stages in that river, it is also necessary to know the discharges that go with them. This, of course, is the primary purpose of discharge data and the first thing to be considered in its study.

The observed values of discharge are plotted as abscissas to the gauge readings at the time of the observation as ordinates, and give what is called the discharge-gauge relation there. They show, as a whole, the increasing discharge capacity of the river from low water up, which may be expressed graphically by a mean curve drawn through the plotted points, or mathematically by an empirical equation. This, called in general simply a discharge curve, is taken for the scale of stage at that location. Some of the discharges may be below this mean curve, some above it. Barring errors of observa-

tion, these are different levels of a given stage, and mark either a change there in the conditions of flow or bed changes.

Of course no scale of stage in any river has an absolutely determined zero; the lowest low-water is as far down as it may be followed. In the same way, whether it is given by a graphic curve or an empiric equation, it has a physical limit at the top of the banks, or the level of overflow; it may in cases be more or less continuous above this level, or it may be altogether different. But between these limits it is a given absolute measure of all the average or normal physical and hydraulic properties of the river at that location.

How this scale is to be marked is still a matter of convenience. At any point in the river it is simply a range of level between low and

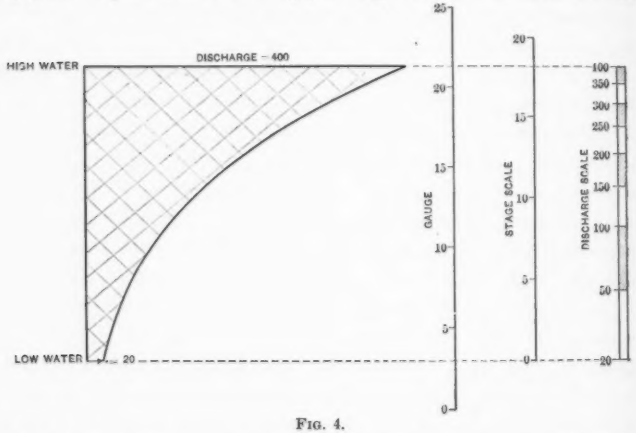

FIG. 4.

high water, with a given range of discharge corresponding to it. Drawn with a general characteristic of the Missouri River, but without any necessary reference to any river whatever, it is shown in Fig. 4, with some of the forms of marking which have been taken in different uses of it.

Where either the gauge or stage scale is used, the normal discharge curve corresponding to it is given at the beginning, and after that the stage is simply taken in feet and fractions with the given values of discharge understood. The discharge scale has the advantage of bringing together the two elements which define the given stage there, but it does not give their coincident values so accurately

everywhere. On it here the discharge is marked in thousands of cubic feet per second, a unit in which it will be generally expressed.

While each of these methods of marking the given stage has, as will be seen, its special field of uses, in the immediate study of the data, the gauge, of course, is the first one that is taken; but from the first, also, this must be clearly distinguished from simple gauge readings, for they have in them as well all the variations which may occur in the surface level of a given stage there through the flood season. It is true that, as this scale of stage differs materially from point to point in the same river, it may also change more or less from time to time at the same point, but what it is must be gotten before such changes can be followed, and it is important, in determining what it is, not to mix it up at first with all the sequence of bed changes and different conditions of flow which follow each other in infinite variety and endless succession from season to season. These have been called simply changes of plane, without regard to their causes, and in what follows they will be so designated.

It may be recognized here that the term "stage" is used, in a highly technical sense, as a standard of measure in rivers. Without a measure for force or for heat but little progress could be made in those sciences, and some definite measure is equally necessary in the hydraulics of alluvial rivers. In its general use, stage refers to that range in a river between high and low water, as high-stage, mid-stage and low-stage, or, in terms of gauge readings, 10, 20 and 30-ft. stages. But these are plainly very different things in a river say 100 ft. wide and another 1 000 ft. wide. This ordinary use expresses but one element of something which has at least another dimension. In terms of slope, area and mean hydraulic radius it has three, with none of them known and no known relations between them; but these are all summed in the coincident value of discharge, which, observed anywhere for one stage, is everywhere on the river for the same stage, the same value.

Expressed in this scale of stage, the different parts of the river may be brought into immediate comparison, and also the different types of different rivers; but as yet it has had no special reference to any particular river, and has hardly touched upon the data from which it is to be determined. The mean curve drawn through the discharge-gauge relation has been noted as giving it, but this alone forms only a

part of the data by which it is determined, and indeed a very small part in its extension over a whole river and the study of its general relations there.

Of course, this scale of stage gets its absolute value finally from discharge observations, but were not the relations in which all the different scales of stage stand to each other in different parts of the river first known, the study of discharge data might very imperfectly determine it; and these relations are almost wholly a question of the gauge data.

Between tributaries, whenever the river is on a stand, it is plain that its surface simply marks its points of equal stage on all the gauges ; or, referring to Fig. 4, the low-water reading on the gauge is everywhere the zero of the stage scale with a corresponding discharge of 20. The top of the flood, in the same way, may mark the equal high-water stages, and intermediate stages are continually being given, between the varying rises and falls, in this direct manner.

In all this it will be found that the different scales of stage in the same river in general stand to each other in a simple ratio. That is, these readings, on any two gauges taken as ordinates and abscissas, give a relation between them which is a straight line. And further, in any rise or fall of the river, when a suitable time is allowed to let the difference of stage pass from the location of the upper gauge to that of the lower, the readings so taken mark the same line through the intermediate and recurrent stages and in all the periods of rising and falling river.

Fig. 5 will serve as a simple illustration of this. On it the Missouri series of discharge observations, taken in 1879 at St. Charles, are shown plotted to their St. Charles gauge readings, with the mean discharge curve given by them there for that season. With this also is given the relation between the gauge readings at St. Charles and those taken at Hermann, 69 miles up the river. In this the time interval of $\frac{3}{4}$ of a day is allowed for equal stages to reach St. Charles from Hermann, and interpolated St. Charles readings, this much later each day, are taken for ordinates to which the Hermann readings are plotted as abscissas.

All the readings on these two gauges, from early in April to the fall low-water in November, are shown in the plotted points of this gauge relation. And it is plain that for all values of discharge corresponding to the St. Charles stages, accurate corresponding Hermann values

may be taken immediately from this line. Or, the gauge relation is said to transfer the St. Charles discharge curve to Hermann, and this it does also with altogether about the same precision with which the curve is originally determined for St. Charles. Thus, in 1879 there are 170 discharge observations at St. Charles to give the mean curve there, while, at the same time, there are somewhat more than 200 entirely independent observations of the surface level at St. Charles and Hermann to fix this relation between them. Either determination has its possible

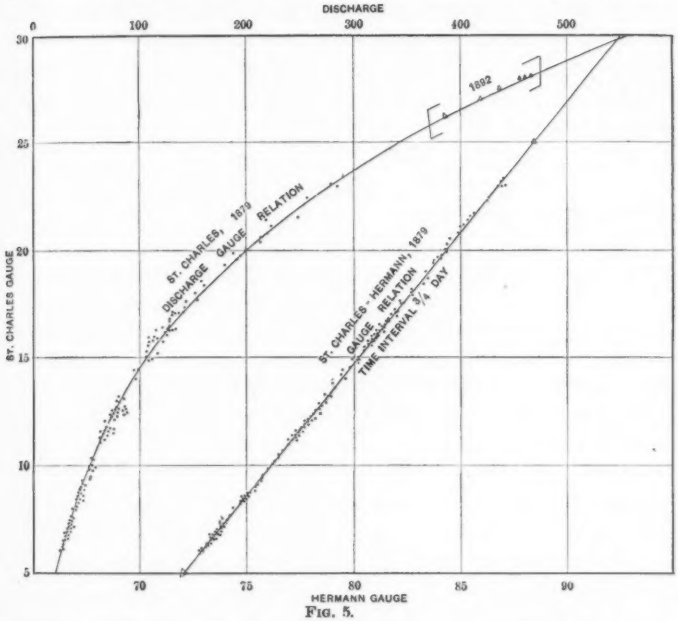

FIG. 5.

error, but the resulting scale of stage at Hermann may not have more but actually less of this error in it than the St. Charles determination.

As the St. Charles discharge curve is transferred to Hermann, so again it may be transferred from gauge to gauge along the river, meeting a positive test of both the original curve and of all the transfers at every other location where discharge observations have been also taken on the river in the same period. The only other observations made in this year on the Missouri were a short series at Sioux City, some 780

miles further up, and a transfer and check over this distance is a long, and necessarily a very careful, process. In passing such tributaries as the Platte or Kaw Rivers special studies of the gauge relations there for years are also necessary, and need either some discharge observations on the tributary, or discharges of the main river above and below it.

In fact, it is only in the same river that the relations between the scales of stage are straight lines, and "the same river" is, of course, to be understood as a physical term, and not a geographical name. The Missouri above and below the Platte need not have been the same river, but essentially it is, and this relation practically holds on the Missouri, at least from Sioux City to its mouth. Indeed, the smaller tributaries further down, in their low-water periods, have not more than 0.1 or 0.2 ft. of effect on the gauges below them, even at the extreme low-water stages of the main river, and this, of course, amounts to little or nothing at the higher stages.

However, a flood out of these tributaries is another matter. The Osage in extreme flood may add something like 150 to the discharge of the main river, and it is plain that in such periods, the line of relation, between the scales of stage above and below it, is no longer given by the gauge readings. The flood does not necessarily affect the scales of stage at either of the locations, unless perhaps the upper one is within the range of back-water action from the tributary, but the marked excess then of the lower gauge readings must show in a corresponding divergence from the line of the ordinary gauge relation, and, in fact, on the Missouri River, this is used as a measure of the floods from the tributaries.

Such cases are, of course, to be met with in plotting the gauge relations, and are excluded in these transfers of the discharge curves. Some of them come at inconvenient times, but long low-water periods are the ordinary conditions of these tributaries, and, using only such periods in general, the transfer may be carried across them without sensible error. In the same connection, also, the investigator will soon find that he has to deal with more or less gauge data which should never have been published. The effect of the tributary flood is continuous, and this fact shows in the gauge relations which follow it; but false readings are not, and by that they are marked at once on the gauges where they are made. The line selected for Fig. 5 was free

from all such complications. It is not at all an exceptional gauge relation, but it must not be assumed that all gauge relations are by any means so simple.

So far, the scales of stage have simply been taken as marked on the gauges, and, whether an original discharge curve, or a discharge curve transferred, they serve only as a measure of a more or less limited reach of the river at the given location. Before considering a more general condition, it is convenient to pass to the second form of marking shown in Fig. 4, and called the stage scale. For this, 5 ft. on the St. Charles gauge is arbitrarily taken as a low-water standard, and 25 ft. as a high-water standard, corresponding in this year to the discharge values of 19.6 and 347, respectively. In Fig. 5 the corresponding levels on the Hermann gauge are shown as determined on the line of that gauge relation. And taking these corresponding points from St. Charles to Kansas City on all the gauge relations between them, in all the combinations in which they may be plotted, gives for this year the equivalent gauge readings called low and high water, respectively, in Table No. 1, at the locations of each of the given gauges.

TABLE No. 1.

Location of gauges.	Mid-Bank Distances.	Year, 1879.		Equal stages (h) (high water).
	Miles above St. Charles.	Equivalent gauge readings.		
		Low water.	High water.	
St. Charles	0.0	5.0	25.0	20.0
Cottleville Landing	16.5	25.2	40.1	14.9
Washington	40.5	46.4	62.3	15.9
Hermann	69.0	72.0	88.5	16.5
Fisher's Landing	87.5	88.9	106.0	17.1
Jefferson City	112.2	110.7	127.5	16.8
Providence	136.5	134.2	150.0	15.8
Boonville	160.4	156.0	172.0	16.0
Glasgow	187.0	178.5	196.5	18.0
Miami	222.5	212.7	230.7	18.0
Waverly	249.0	233.7	251.0	17.3
Lexington	270.0	253.7	272.8	19.1
Camden	283.5	267.0	283.0	16.0
Missouri City	307.0	285.0	302.7	17.7
Kansas City	332.7	307.0	325.4	18.4

The differences between these arbitrary high and low waters, called equal stages, mark, of course, the scales of stage at each of these locations, mid-stage being simply the half of these values, and so on, for all intermediate stages. But the fact that these are 20.0 ft.

at St. Charles, and only 14.9 ft. at Cottleville Landing, 16.5 miles up the river, or 19.1 ft. at Lexington, with 16 ft. at Camden, 13.5 miles above it, shows distinctly the marked longitudinal variation which may be found in the river; and gives, at the same time, its definite measure. The whole length of the river is certainly a sequence of such variations, in its extremes at least as large as the differences found in these cases.

This sequence, in its actual longitudinal form at any time, has not yet been determined. But, while in certain bends it may stand unchanged for a long time, in general, it is not fixed. What was 20 ft. at St. Charles, in 1879, was but 18 ft. some five years later; in which case the river there had left its old channel around the St. Charles Island, and shortened its course by several miles. In the same way, the 16 ft. at Camden, in time, may take the place of the 19.1 ft. at Lexington. And this sequence, in its time variation, may be followed with precision in the discharge and gauge data. It is one of the hasty conclusions of a superficial study of this matter, that there are no gauge relations because they are not fixed ones, while, on the contrary, it would certainly be very surprising to find such a longitudinal variation in the river actually tied to all its gauges.

However, in the Missouri in general, and it is inferred in other northern rivers, there is really a period in which there are no such relations. Of course, when the river is frozen over there is not much rise or fall to mark any relation, but it seems that an ice-bound surface may result in a different distribution of slope from that of a free surface, and, in the place of the river standing at the same level on all the gauges, it may, in such a case, change in a perfectly arbitrary way at any of them. The clearing out of the ice also probably cuts up its bed pretty badly, as the forming and breaking of ice gorges might be expected to do. But certainly during and immediately following this period no relations between gauge readings may be found. That it soon readjusts itself and all its flow, in all the variations along its course, and, in all its changes from high to low water, is again found held in this chain of simple stage ratios, is not at all inconsistent with what has preceded it, and on the whole marks no less but rather more the presence of a determined and controlled equilibrium.

And, finally, the larger changes noted in the equal stages, which are in general a matter of years, must also be supplemented by

smaller changes which may be only a matter of seasons. Not only may the river change its whole course and its whole form at the location, but the bar may be built further down and bring the gauge from the level of a lower into that of an upper pool. The first may have an effect of several feet, the second may be limited to fractions of a foot, but on the Missouri the second may also mark several distinct gauge relations in the season, or throw a part of the plotted period out of the line of the general relation. These, of course, mean different scales of stage there for each period, but in such details the scales of stage are certainly restricted to very limited reaches. This might be of value in special studies, but, as a general river measure, the average is all that is wanted; and where there are two or more such changes in the season, a mean is taken for the gauge relation.

This outline will show the general processes and results of a study of discharge and gauge data. It gives an absolute measure of the river at a number of different locations, which probably cover all its variations, and follows with precision all its changes at each of them. To close for the Missouri, it is only necessary to add the fact that the Missouri in different years is a different river. Or, to state this technically, the level of the zero of its general stage scale is found to change materially between a sequence of high-flood and low-flood seasons.

Taking, for instance, the fall low-water of 1883, following a succession of phenomenally high-flood years, the low-water level corresponding to the discharge of 19.6, in the place of being the gauge readings from 5.0 at St. Charles to 307.0 at Kansas City, given in Table No. 1, would be found to average about 1 ft. lower. Whether or not the high-water stage is affected in this is still unsettled; from such data as there are on the subject, and from the nature of the change, it is inferred that it is not; in which case all the values in the column of equal stages would then average 1 ft. more. In the same way, the fall low-water of 1895, following a succession of phenomenally low-flood years, is about 1 ft. higher, giving values in the column of equal stages averaging about 1 ft. less.

The process of such changes will show later. They arise from the counter-actions, already noted, of erosion which would fill up the bed, and of that lateral resultant which would keep it down. Every cutting bank and bar is caving into the river continually, in

this way throwing upon the bed material which is not carried away, for there is no evidence of an increasing movement of material down stream; and indeed, accumulating as it would, it could not be thus taken up in the flow without soon overloading it; therefore, it must simply be spread over the bottom in some way, and this, at all times,

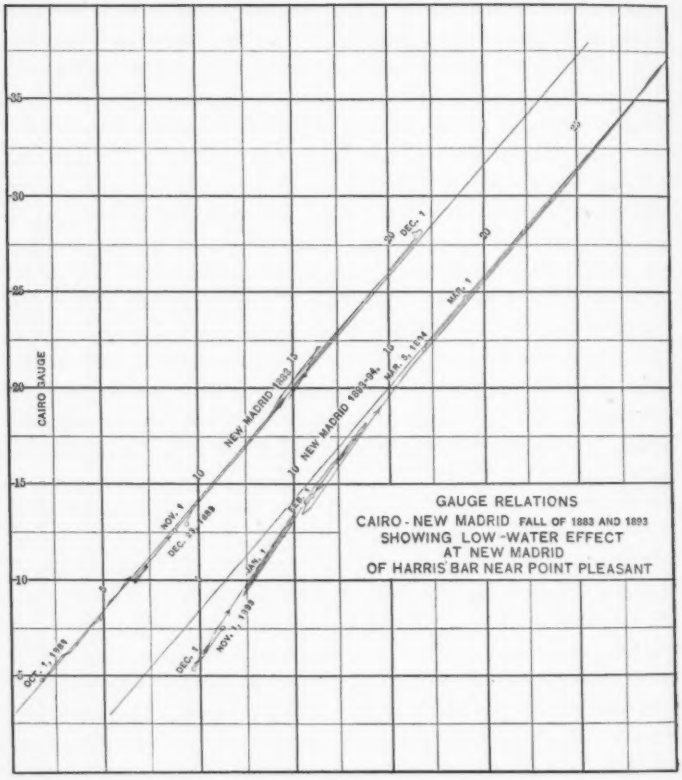

FIG. 6.

below the existing water line. It is really, then, only the flood which can move this material up on the slopes of points and bars; and the extreme flood is required to throw any of it altogether out at the bank level. Thus there is no doubt that the action of the series of extreme floods is this general clearing out of the low-water bed, and of the

series of low floods a corresponding filling up, notwithstanding the fact that this has not yet been observed in anything but these stage measures.

With all that has been given in this study of the Missouri, the subject may be followed now into the data of the Lower Mississippi; and it will be enough to note briefly the special features which the given processes show for this river, and at the same time any new problems which its marked physical difference and greatly enlarged scale may

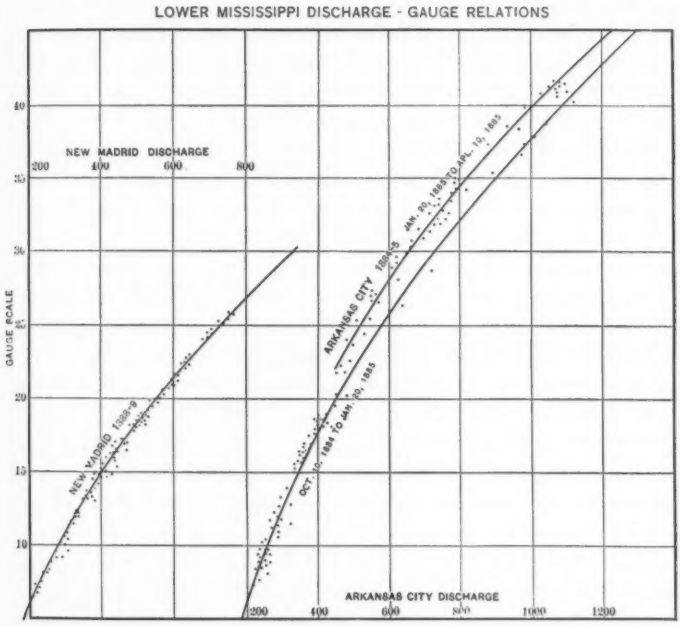

FIG. 7

bring into view. And, certainly, between a river of the size of the Missouri, and one whose floods have some three times the extreme discharge and twice the rise, this matter should be fairly covered for alluvial rivers.

Figs. 6 and 7 serve as illustrations. The New Madrid discharge-gauge relation in Fig. 7 does not differ essentially from that of St. Charles on the Missouri, and it may be seen by the Cairo-New Madrid gauge relations in Fig. 6, that, in general, it may be transferred in just

the same way in which St. Charles was transferred to Hermann. But it will be noted here also that, while the relation at the higher stages has not changed perceptibly in 10 years, a decided but limited range of bar effect has come into the surface levels of the New Madrid low-waters. And it is this pronounced stability of these relations, some of which now go back nearly 50 years, subject now and then to these low-water variations, disappearing between the 10-ft. and the 20-ft. stages, which may in general be said to be the special features of the Lower Mississippi gauge relations.

This low-water form, however, is not a variable but a fixed condition in the 300 miles between Red River and the Gulf. Even as high up as Red River, the Gulf level is some 10 to 20 ft. above the mean bottom of the Mississippi, and through all this reach it acts upon the low-water surface in much the same way as did the crest of the bar in 1893 at New Madrid. For any such reach, of course, the variation of the scale of stage through it is a special and a single problem in each case.

But the marked exception of the Lower Mississippi gauge relations to the general features noted above, is found since the times, and at the locations, in which its great floods have been held within levees. It began to be seen, more than 10 years ago, along the swamp fronts, where the large natural outflow was being materially checked by the building of levees, that a general lowering of the low-water level was an accompanying phenomenon; and it was soon inferred that the levees there, producing abnormally large floods, like the series from 1881 to 1883 on the Missouri, were simply clearing out the low-water bed in the same process. Indeed, it was here that the action of that lateral resultant of the movement of material in a river was first suggested, even before the magnitude of erosion, determined between hydrographic surveys, had made it so plainly a necessity, and before the reverse process had been tested in the filling up of the Missouri River bed after the succession of low floods in 1895. Simply in these levee effects was it seen that, whatever might be the longitudinal movements in the building up of the bars at high water, it was, after all, the magnitude of the flood volume which kept down the level of the low-water plane; and the somewhat homely illustration suggested itself that, as a whole, the flood went through the river somewhat like a plow.

All these features of the Mississippi, however, raise no essentially new questions; but, the Arkansas City discharges, in Fig. 7, do. The

marked difference in the level of the discharges between the first and second periods is a change of plane of a character which must be considered here. If it was simply a permanent local change at that point, it would naturally be assigned to some radical change in the regimen of the reach, similar to that noted at St. Charles, when the range of stage there changed from 20 ft. to 18 ft. But, far from being such a permanent change, it is not even a local one.

As may be seen in the example given, the gauge relations on the Mississippi are not only especially stable, but they are also, in general, especially well-defined single lines; and, in that case, every variation which shows in the surface levels of the same discharge at one of the gauges, must show in the given ratio and the given period at the other. Changes of plane are transferred as well as discharge curves, and, as far as the gauge relations below Arkansas City, each shows a single line for both of these periods, this change of plane is necessarily an identity all the way down.

It is, of course, only actually identified at the location of the lower gauges; but where it is found repeating itself in this way, in both its ratio and period from gauge to gauge down, there can be little doubt that it is simply continuous between them. And in this process the change of plane shown at Arkansas City, is found to be a continuous one as far down as it can be tested accurately, and at least, in this case, for over 300 miles.

This change of plane coincides roughly with a flood out of the Arkansas River, it does not show on gauges above, and it is practically continuous through the rest of the flood and as far down as it can be tested through the rest of the river. While such a change of plane certainly holds a difference in the general slope from above, this difference does not account for it, for this difference is the thing itself. In the same way the difference of slope between the rising and falling river does not help matters, for this difference is as great at Helena as it is at Arkansas City. And, in fact, this change of plane is found here, as repeatedly elsewhere, to have much the same magnitude between the two periods whether the river is rising, falling or stationary in either of them. Indeed, as was pointed out at first, not much dependence is to be put in the direct slope-effect in the flow of rivers; it appears and disappears continually, as a general slope controls or does not control the flow.

Such a continuous change of plane, in the case in hand, has been considered, probably in part at least, a change in the conditions of flow accompanying the flood out of the Arkansas. And, for the entrance of floods from tributaries, there is this to be said, they certainly raise the surface level of the river above them without any corresponding increase of discharge. In cases in the Lower Mississippi this may be followed up with precision on the gauge relations; thus, floods out of the Arkansas River have been seen distinctly to raise the surface level of the Mississippi over 40 miles above by more than 1 ft. The flow of the main river, then approaching the tributary with its momentum somewhat reduced at a continually increasing rate, might, perhaps, be expected to carry a higher plane below for some time and some distance. And, indeed, in such a case as the run-out of the Lower Mississippi from Red River to the Gulf, where, even in the highest flood, only a small fraction of the water in the river is above the Gulf level, and the great mass of it has not a foot-pound of potential energy, such effects of floods from Red River certainly seem to have startling proportions.

In the years of great floods, also, changes of plane of this character, are marked, from Cairo down, and, as changes in the conditions of flow, would be assigned to the large return flows and back-water drainage following the extended overflow. And, as they do not show here to any great extent in the low-flood years like that of the New Madrid discharges given, some such assumption is not altogether unwarranted. But, at the same time, it should be recognized clearly that marked bed changes also occur between these periods. The raise to the top of the flood has the effect of the erosion through the greater part of a season at its surface, with all the bar-building forces of the river under it, while this is practically reversed in the fall. And, until this movement of bed may be determined, it is needless to discuss further the special causes of changes of plane of this character.

Of course, with such changes of plane in the Mississippi, there is little doubt that changes of plane of the same character occur in the Missouri, but, if they are not large enough to be seen there, they need not be considered. The same change in the conditions of flow which would change the surface level by 1 ft. in a river from 50 to 100 ft. deep, would only change it by 0.1 ft. in a river from 5 to 10 ft. deep; and in the different type, the reduced scale, and much more active bed

changes of the Missouri, changes of plane of this character are simply unimportant phenomena. In the same way, again, if such studies were carried into a river like the Platte, all the importance of the Missouri River gauge relation might disappear in the small range and continuous bed changes of that river. Hydraulics is limited to relations which show; and perhaps it is altogether possible to study with a microscope, the flow in the simplest form of flume, in such detail as never to understand it.

Finally, all these detailed studies of the discharge and gauge data in a river may be put together in the discharge scales, and show at a glance all the floods of a river for years, with the combinations which make up such a flood as that of the Lower Mississippi.

Figs. 8 and 9 illustrate the use of the discharge scales. The regular gauge readings are plotted from day to day as ordinary hydrographs, and the discharge scales are drawn through the flood season from low-water up. Arranging these, then, in order down the river, all the contributions from the tributaries show in the differences between the flood volumes from gauge to gauge, and may be checked readily at any point with summations of the tributary increments taken from the gauge relations, while the combined floods of two rivers, such as the Missouri and Upper Mississippi, are seen summed up in the flood volumes of the Middle Mississippi at St. Louis.

In these diagrams simply the average discharge scales have been used, and no account has been taken of changes of plane through the period, but, of course, in checking up the final discharge curves in the different rivers it is necessary to take account of these changes. And there, indeed, the whole study of these data in the rivers meets one of its severest tests. If the sum of the St. Charles discharge and the Grafton discharge do not make the St. Louis discharge there is something wrong. Individual discharge observations may have their errors, but correct scale of stage can only have changes of plane and wrong gauge readings.

All this, however, may be followed, well enough for an illustration, in the average scales in Fig. 8. Thus, the top of the 1881 flood is a little less than 500 at St. Charles with a little less than 350 at Grafton, while St. Louis shows about 800. But the plane at Grafton is raised by an up-stream effect from the extreme flood in the Missouri, Grafton discharge observations at that time showing this distinctly, and, allow-

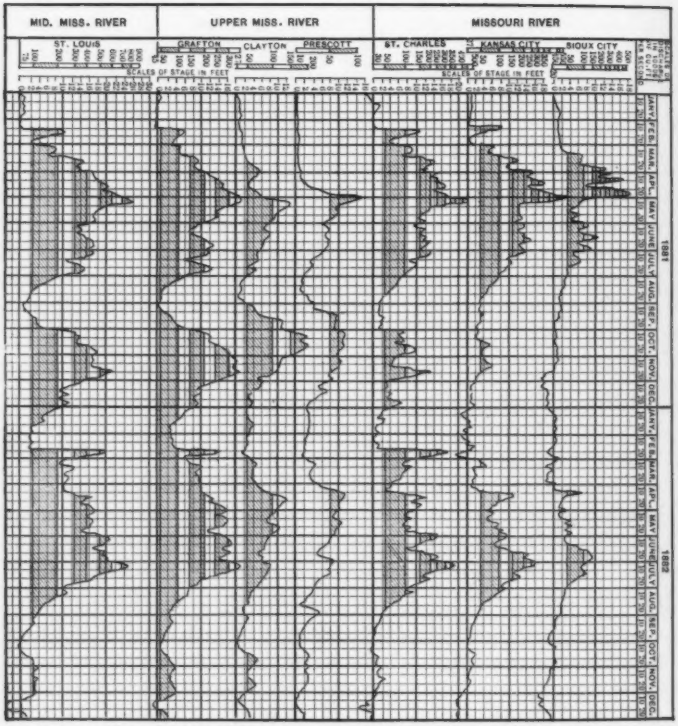
MID. MISS. RIVER
UPPER MISS. RIVER
MISSOURI RIVER
ST. LOUIS
GRAFTON
CLAYTON
PRESCOTT
ST. CHARLES
KANSAS CITY
SIOUX CITY
SCALES OF STAGE IN FEET
1881
1882
JANY.
FEB.
MAR.
APL.
MAY
JUNE
JULY
AUG.
SEP.
OCT.
NOV.
DEC.

Fig. 8.

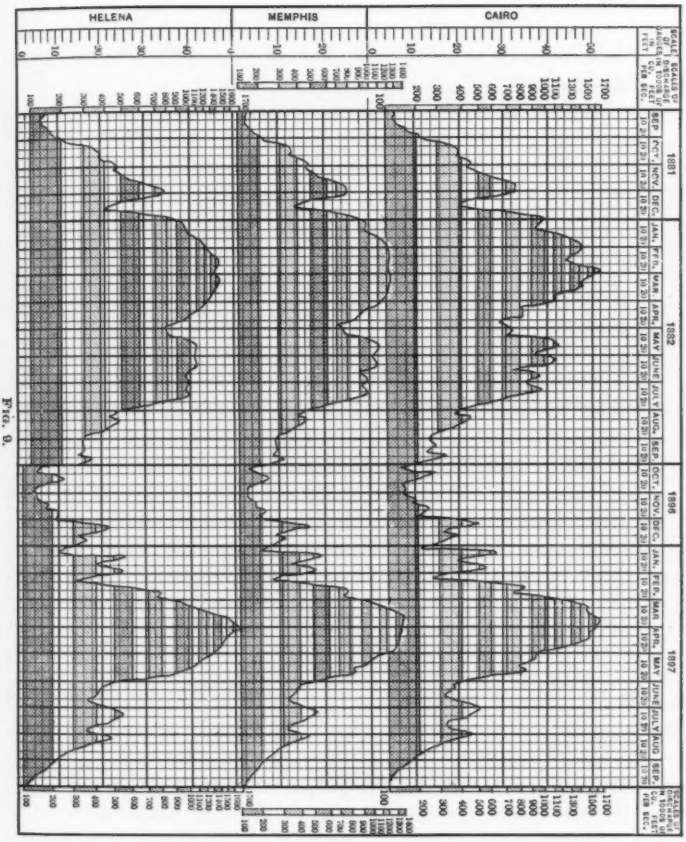

FIG. 9.

ing for this, it checks very well with St. Louis and the discharge from the Missouri. The next low water is 50 at Grafton, and a little below 50 at St. Charles, differing about the same amount from the 100 at St. Louis. And again, the top of the 1882 flood is 350 + at St. Charles, and 350 + at Grafton, with 700 + at St. Louis. At the same time, the irregularity of the ice period in these rivers is marked, and particularly in the forming and breaking of ice gorges from Sioux City down, in the spring of 1881.

Fig. 9 gives, in the same way, from Cairo to Helena, two of the greatest floods in the lower Mississippi; and Fig. 10 is an outline map showing the river's system of swamp basins. In general, all along these basins the ground slopes back from the river into an area of low swamps threaded with lakes and bayous, and draining into such rivers as the St. Francis and Yazoo.

As the Mississippi rises above its bank-full stage, the flood water is drawn off into these upper swamps, to be returned to the river at their lower ends after an interval. Fig. 9 shows this action through the first or St. Francis basin. The total volume of the flood, not only given by the Mississippi stages, but also traced down from its sources through the combinations of the upper rivers, is shown in the discharge scale at Cairo, while the same flood is shown at Memphis at about its point of maximum outflow, and again at Helena, where it receives the return flow from the St. Francis basin. However, the total return flow is not shown at Helena, since the outflow into the upper Yazoo at the same time is necessarily lost to its stage and discharge.

The former condition of free overflow is shown in the year 1882, and the first effect of the levee system now in progress in that basin is shown in 1897. And though the system was far from complete at the time, and was badly broken in the flood period, still the increase in stage and discharge is most marked. While the flood extreme of 1 700 at Cairo in 1882 is reduced by outflow to but little more than 1 200 at Memphis, and comes back with inflow to about 1 400 at Helena, in 1897 it reached about 1 400 at Memphis, before the breaks in the levee relieved it, and came back again at Helena to the Cairo value of 1 700.

This will illustrate the method of handling the extreme-flood data, which, as before noted, lie beyond the range of regular discharge curves. While not pretending to represent the actual discharge at any spe-

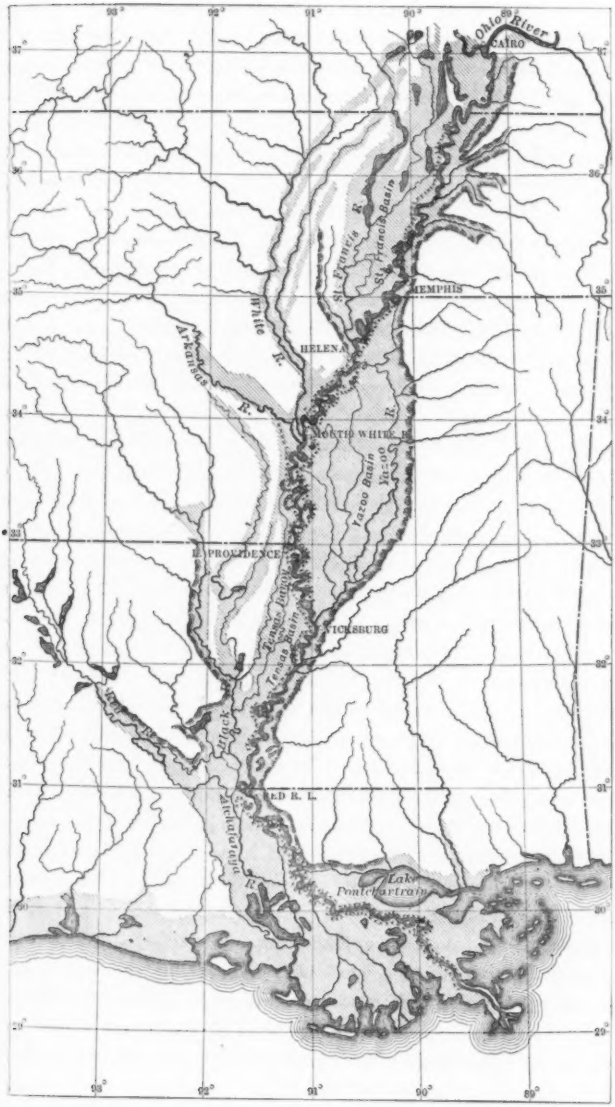
Ohio River
CAIRO
MEMPHIS
HELENA
NORTH WHITE R.
VICKSBURG
RED R. L.
Lake Pontchartrain
St. Francis Basin
Yazoo Basin
White R.
Arkansas R.
Tensas Basin
Atchafalaya R.
Mississippi R.

FIG. 10.

cial point in the flood, these scales, as an average, bring all the data together and compare their total volumes in a form in which some reasonable consistency may be recognized. However, further down the Mississippi, and especially since the construction of levees, the discharges are often so irregular, between the draw of a large crevasse on the one hand and the lake-like character of points in the general overflow on the other, that they can hardly be followed through the flood period, even in this system of rough averages.

Finally, Cairo and Memphis, in Fig. 9, show very well the fixed character of the general Mississippi regimen. The 15 years from 1882 to 1897, has brought no marked change there; while at Helena, in the same period, a case of the levee effect before noted is given. Here, in 1897, a glance shows that the discharge, as brought down to Cairo, from the rivers above, and as found repeated at Memphis, cannot be continuously some 50 less at Helena, as the 1882 discharge scale would give it. The precise lowering there is a matter of the gauge relation; but nowhere is the general fact more plainly seen than on the discharge scales.

In following the discharges down from Cairo to Helena through this period within banks, their readings on the discharge scales are essentially the same as they should be; for, in general, the tributary increments here would not show on these scales. Thus, the crest at the end of November, 1881, is a little less than 800 from Cairo to Helena, followed by a fall to just 400, and a rise to about 1 000, where the limits of overflow begin to show. But it must be noted also, that this does not mean that the actual discharges for the same scale readings in different periods are necessarily the same. The scale readings are normal values, and, so long as the actual discharges differ from the normal by the same amount all the way down, their normal readings will be the same.

This general identity from Cairo to Helena, therefore, is not only an identity of the discharge, but also an identity of plane. That continuous plane, noted in the case of the Arkansas City discharges, shows all through the data of the Lower Mississippi. It is referred to here again in order to note finally that whatever its causes may be, it has, from point to point down, exactly the same flood sequence preceding it, and its accurate recurrence from gauge to gauge only marks it as a very general and a very definite phenomenon; it does not add essentially to the difficulty in its explanation.

Flood Movement.

So far, it has simply been noted, in connection with the gauge relations, that, when the river was rising or falling, a suitable time was to be allowed for the given stage to pass from the location of the upper gauge to that of the lower. The precise determination of this time, from the gauge readings themselves, forms the subject of this study of flood movement.

In the rough, this time interval is seen very well in the hydrographs of any river simply by noting the difference in the times at which the flood crests show in order down on their respective gauges, and it is not only marked at the crest, but, somewhat less distinctly, at the foot of each rise. Thus, in Fig. 8, in the period May-June, 1881, on the Missouri, the general interval of about seven days from Sioux City to St. Charles is fairly indicated in the time between these points marked by the various crests and hollows; though it may be more or less blurred in the case of any one of them by the changes of plane and tributary increments which show in this period. In the same way the special retardation at the top of the April flood of that year will be noted. And, passing to the Upper Mississippi, its much slower rate of flood movement is shown most distinctly. Thus, comparing the movement from Prescott to Clayton with that from Sioux City to St. Charles, in the Mississippi, the flood takes about the same time to pass over about one-fourth the distance.

In the Lower Mississippi, Fig. 9 shows an interval of some three or four days from Cairo to Helena, and if any one should undertake to follow up this interval on the discharge scales, through all the rises and falls of the river, by noting the difference in time between the same discharges at Cairo and Helena, he would be struck at once with the general accuracy and the constancy of this interval, not only for the crests and hollows, but also for all the floods within banks.

But days are far from being the measure of precision which is wanted, and to obtain this, very different methods are needed. It is evident, on the face of it, that the time required for a given stage to pass from an upper to a lower gauge is determined with the greater precision the greater the difference of stage which is passing. Thus, the periods of rapid rise or fall are those which must be used for precision, while the period is hardly defined at all in the slower changes

of stage at the flood crests; though, taking these crests as a whole, symmetrically, it may still be seen in them. Almost any interval at the crest will give the gauge relations, while the line between any two gauges is given without any interval at all through all the stationary periods of the river which extend to both of them; but where a rise or fall of some 2 ft. a day occurs, an error of a tenth of a day in an assumed interval begins to make distinct divergences from the line of the gauge relation.

Having the line of the gauge relation given, and plotting such rises and falls with assumed trial intervals, the divergence of each plotted point from this line, of course, marks the error in the time assumption at that stage; but as, in the great mass of the data, the gauge relation and the time interval have to be determined together, it is convenient to have in mind, clearly, from the first, certain standard types which different trial intervals must give to the form of such a plotted relation.

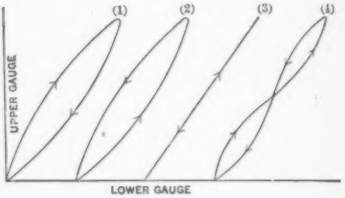

FIG. 11.

Types of trial time intervals are shown in Fig. 11. The first is an interval altogether too short. If, in plotting a trial relation, the assumed interval was less than the true time, then, on the rise, the lower gauge values taken would not have had time to rise to their equal stages, and would, therefore, be too small in proportion to the rapidity of the rise. In the same way, on the falling stage, the lower gauge values would not have had time to fall enough, and would, therefore, be too large in proportion to the rapidity of the fall. The trial relation would, therefore, plot as in (1) where the rising and falling stages are indicated by the directions of the arrows. In the same way, if the trial interval for the flood is taken too long, the lower gauge values would be too large for the rising and too small for the falling stages, giving the trial relation shown in (2). Between (1) the interval altogether too short, and (2) the interval altogether too long, the relation of the flood can always be reduced to one or the other of the forms (3) and (4). The form (3) shows very plainly that this interval for the whole flood is a constant, or that the flood movement does not vary with the stage; while all conditions of a variable

flood movement are included in the form (4). And here, in the special case indicated by the arrows, it represents a flood movement which increases about uniformly with the stage; that is, the mean interval is about as much too short for the lower part of the flood as it is too long for the upper part.

By more or less trial plotting of selected floods, the character of their movement, between any two gauges, may be recognized, and, in case the interval is a constant, it may generally be determined in this way to the nearest tenth of a day. It is also plain, however, that any flood out of a tributary entering between the gauges in this period would destroy altogether the simple form of the relation shown in (3). Indeed, unless such periods were recognized and excluded from the study, they would lead to very serious errors in both the rate and the character of the flood movement.

It is also necessary to allow for those changes of plane which may occur from time to time at the location of one or the other of the gauges. To determine the time required by a given stage to pass through the reach it must be taken on the level which it has in passing. So long as the levels of the given stage remain the same at the two ends, or change together, as they do through those continuous changes of plane noted on the Lower Mississippi, the relation will give the form (3) when the correct time interval is taken, but when the level of the given stage changes at one of the gauges without a corresponding change at the other, the form of the true gauge relation should shift in the same way; and an interval which might make it approach more closely to the form (3) would not then be a more correct interval, but an erroneous one.

These, of course, are the same variations which have been met in the transfer of discharge curves, and which, with a little care, do not affect sensibly the accuracy of that process. But here, in the determination of an element which is marked alone in the form of special periods of rise and fall, they are much more serious difficulties. Thus, taking gauges above and below a tributary, after getting by trial a correct interval between them, and then, with this interval, systematically plotting their relation through years, a number of the floods might be seen to correspond very closely with the form (3), and a number of others might be marked clearly as affected by floods from the tributary, or changes of plane, and rejected. Between these two,

however, there would still be a number of periods in which it would be uncertain whether their form did not measure as truly a flood movement there as did the form (3), and in which, if they did, it would be plain that the flood movement, in that case, was not at the same rate as the other, nor indeed any longer a constant in its character.

This, of course, would have been all well enough if it had proved that the flood movement varied from time to time and for different forms of floods, but when the precision of the determination is also considered it is at once seen that in itself it simply does not prove anything. There is no use in taking interpolated values between the daily gauge readings any closer than to the nearest tenth of a day; if for no other reason, because it is doubtful whether the observers have actually read their gauges closer to the set time, if indeed they can be counted on for that. With an interval then of some two or three tenths of a day between the given gauges, this relation showing in any case the form (3) means only that the flood movement at that time is this given rate and is constant within the limits of some $\pm$ 20%, while in the other case it means that its rate is something more or less and variable within the same limits. Thus, the two cases simply are not precise enough for their contradictions to mean anything.

Again, while the short reaches lack precision in this time element, the long reaches lack precision in the equal stages. When the river is rising or falling 2 ft. a day, a change of 0.2 ft. in the stage is equal to a tenth of a day in the time interval, while in periods of less rapid rise and fall the time equivalent is correspondingly greater. In a long reach, therefore, covering a number of tributaries, the summations of their increments, small in themselves, may affect materially the apparent time of the passage of a given stage through the reach; and even where they may not do this, their combinations tend to curtail greatly the periods which are practically free from this difficulty, until finally, it is not once in years that a suitable period for the determination of its flood movement can be found; that is, a period without change of plane between the gauges, all the tributaries being at a fixed low-water, with a sharp rise and fall of the main river.

To meet all these difficulties in the precise determination of the flood movement, the system of plotting the data, called extension-gauge relations, was planned. Its principle consists of studying a

flood movement in the form of a number of gauge relations, between the same upper gauge in each case, and lower gauges following in order down; showing, in close succession, the movement of the same flood from the same start, through a series of successively lengthening reaches. The form of each gauge relation, then, in its order, includes all the flood movement of the relations preceding it, with such additional movement as the flood may have in that case between its lower gauge and the gauge next above it.

An instance from the Lower Mississippi extension-gauge relations is given in Fig. 12, covering a reach of 277 miles of that river, from Cairo down. In all this the time intervals have been determined first by trial. Cairo gauge readings are taken uniformly as the ordinates,

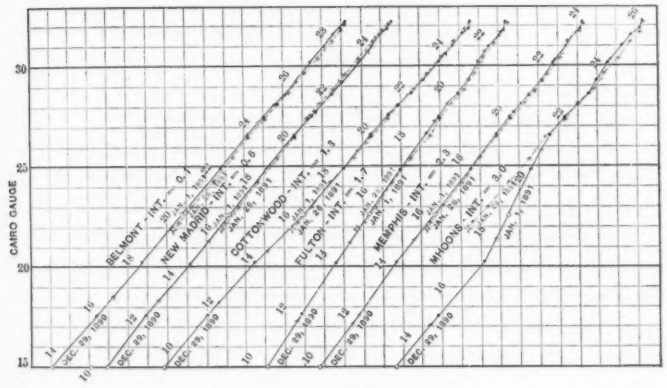

Fig. 12.

and interpolated readings, later by the intervals, respectively, as abscissas, and the gauge relations, Cairo-Belmont, Cairo-New Madrid and so on, to Cairo-Mhoons, are plotted in succession. Altogether, they leave no doubt whatever as to the movement of this flood. The Cairo form is simply reproduced in a given ratio on each of the gauges after the lapse of a fixed time.

Gauge readings, of course, contain a tenth or so of accidental error, and no single one of these relations reproduces exactly the Cairo form in the given ratio at the location of its lower gauge; but when the divergence is in one direction at one gauge, and in an opposite direction at the next, there is certainly no change of form shown in

the passage of the flood through the reach; and, indeed, in this case it is repeated even more exactly at Memphis, 230 miles below Cairo, than at Cottonwood 123 miles below, or Belmont only 21 miles below.

Thus, it is not necessary, in every case, to see this flood movement extending down to Memphis, or to Mhoons, in order to recognize it. If it is seen extending down to Cottonwood, and if, from there on, more or less increment from tributaries comes in to break it up, it is still seen, and the reason that it does not show further down is recognized. Thus, a part of the rise at Fulton may be marked with an excess of Fulton stages, and in that case the same part at Memphis and Mhoons would at least be marked with the same excess, though more probably it would actually show increments somewhat greater. Through all this, however, there would be little doubt that the same flood movement was still there, if outside conditions had only left the same flood in the river to show it.

The Mhoons' relation is given here as an instance which needs a special notice. On January 1st (January 4th at Mhoons), the gauge reader reported that his gauge was washed out, and that readings were taken on temporary gauges. This continued until May 23d when his gauge was reset, the inspection showing that his readings were 2.2 ft. too low. It is very evident from the relation that this error was made at the first setting of his temporary gauge, and if it is so taken the flood movement is shown to be very fairly continuous to Mhoons. But if, on the other hand, the error had been distributed over the whole period up to the inspection, as is often done before the gauge readings are published, it would have given an altogether incomprehensible surface movement of the river there for about five months. This is simply a caution which should be borne in mind continually in any use of the gauge data.

It is this process of recognizing, in the extension-gauge relations, what does not, as well as what does, show the flood movement, which gives all the certainty to its determination from the ordinary gauge data. The method may be said to be almost mechanical. The intervals of the various reaches are determined by trial. Stages between the readings of the lower gauges are then interpolated by these intervals, and are each, for the given day, the effect of the given reading on the gauge at the head of the reach, in so far as this has been

determined approximately by trial. These are then plotted in succession, showing the whole surface movement over hundreds of miles, everywhere as nearly in the one relation of cause and effect as a single time adjustment will give it; and all, for the better part of a year, are brought finally under the eye at once, and in quick succession a number of years can be examined. After that the whole matter is simply one of intelligent seeing.

Such a flood movement as that of Fig. 12 is seen frequently and through long periods, extending over the shorter reaches, but with little precision in the time intervals; while, again, through the long reaches, with the much smaller percentage of error in the intervals, the flood movement can be followed only through shorter or less frequent periods. But as far as it is so recognized and followed, so far it is determined. Its actual rate is not given precisely between any of the gauges, but it is there, or it would not be found precisely in their summations of it.

In the case given, the movement may be said to be recognized down to Mhoons, and the interval there determined to the nearest tenth of a day, or to within ± 0.05 in a total interval of 3 days. This is less than 2%; and again, where the same movement is recognized down to Helena, or through a total interval of $3\frac{1}{2}$ days, the limit of error is then reduced to less than 1.5%; and noting repeatedly just such a flood movement in different floods and through all the different stages within banks, it comes to be held as a very general phenomenon of the river, observed with a good deal of precision, and certainly determined more closely than might have been expected simply with interpolations between daily gauge readings.

This, however, is all a case of the constant interval, or the flood movement which does not vary with stage. Type (4), Fig. 11, or the flood movement which varies with stage, is recognized and its form is followed in the extension-gauge relations in much the same way. That is, as the further a flood goes which does not change its form, the more certainly is it seen that its form is unchanging; so, in the case of a flood which changes its form, the further it goes the more distinctly this shows itself in its greater divergences from the standard form at the head of the reach. Thus if Type (4), Fig. 11, showed a perfectly uniform increase of flood movement with stage for 100 miles, in 200 miles it should everywhere show just twice the divergence.

However, the case of a variable flood movement is never quite so simple, for it represents a condition which cannot continue indefinitely. Thus, the upper part of a flood cannot go on moving much faster than the lower part without coming to a point where it would topple over; and, before it reached this, any uniform or normal movement which it might have at a given stage would certainly be modified by the abnormal form which the flood was taking.

It is this fact which makes the determination of the flood movement in the Missouri, from the ordinary gauge data, so intricate. In the Missouri it has probably as definite a normal character as that of the Lower Mississippi, but this, in the Missouri, is a very decided increase with stage, and the normal movement, therefore, is modified at least by limiting forms of the floods, and is subject to variations with their changes. This, with the fact that interpolations between daily gauge readings are necessarily a less precise surface measure in its short and sharp floods, and that its tributaries through long reaches have a much larger proportionate effect on the main river, all taken together, deferred anything like the setting of values for the normal flood movement in the Missouri to a much more advanced stage of the study.

It is, however, in this contrast that it should, perhaps, be noted that a given flood movement in a river is not a law but a fact. Even with the general character of the constant interval on the Lower Mississippi, there are still a few specially sharp floods and some short periods in the transitions from stationary to flood conditions which seem to be exceptions. But that does not detract at all from the significance of the fact that its whole surface movements from low water up for almost its whole time are certainly held in this given fixed relation.

As simply an empiric constant, running through the infinitely complex and ever-changing conditions of flow, this recurrent flood movement seemed to be the one thing in that river which did not change; but why it should be of the same rate at high water as at low, while the mean velocity of the river for the same change about doubled its value, or why it should be over 400 miles a day from Baton Rouge to Carrollton when even the high-water velocities there were less than 100, were questions at the time well calculated to puzzle anyone; for probably everyone would first try to explain this flood movement by these observed velocities.

Such considerations may have some value in finally knocking out of the investigator himself any remnant of a delusion that water moves in threads and filaments; but as these considerations lead to nothing in the matter in hand they are here omitted. This flood movement is simply taken as it is, a well-determined fact of the river, which, as a fact, naturally rests on some property of the river as a whole which is altogether as general and as strikingly fixed as the flood movement. Anyone is, of course, at liberty to try and find this property in observed velocities if the total independence between the two, in the cases just cited, does not satisfy him.

Yet, with all the river absolutely determined in its scales of stage, and such a property of the river as disclosed by this flood movement also given, it seems certain that somewhere there is to be found some equally absolute relation between them. But seeing this and getting the relation are quite different matters, and it was not until the writer had studied analytically the case of flood movements in a chain of lakes or reservoirs* that he began to be able to calculate some of the elements of a river from its given discharge curves and flood movements. The methods at first were crude and laborious in the extreme, and need not be considered here further than to note the final step in this chain of investigation, for from here on the question will be taken up as an immediate deduction of a fundamental physical equation of rivers, with simply examples of its application.

Mathematical Analysis and Computed Regimens.

It is plain, of course, that for a given difference in discharge between the upper and lower ends of any reach, the rate of rise or fall in the reach is determined solely by the area of its water surface at that time. This general relation is given by the equation.

$$(Q_1 - Q_2)\, d\, T = A\, (d\, h) \ldots\ldots\ldots\

river, the special problem is to express the terms in this condition in elements which may be determined independently.

Now, in whatever condition of rise or fall any river may be, $\left(\frac{d\,Q}{d\,h}\right)$ is the rate of change of discharge with its change of stage, and $\left(\frac{d\,h}{d\,T}\right)$ is the rate of change of stage at any point of its flood oscillations. The change in the value of discharge in an assumed interval $\Delta\,T$ is then $\left(\frac{d\,Q}{d\,h}\right)\left(\frac{d\,h}{d\,T}\right)\Delta\,T$. Now, whatever this may be on either rise or fall, there is some point down stream at a distance $\Delta\,l$ where the value of discharge at the end of the interval $\Delta\,T$ is just what it was at the upper point at the beginning of the interval. The difference in discharge between the upper and lower ends of the reach, of length $\Delta\,l$, is then $\left(\frac{d\,Q}{d\,h}\right)\left(\frac{d\,h}{d\,T}\right)\Delta\,T$; and the surface area of this reach is $W\,\Delta\,l$, where W is its mean width in feet. Substituting these, respectively, for $Q_1 - Q_2$ and A in equation I gives

$$\left[\left(\frac{d\,Q}{d\,h}\right)\left(\frac{d\,h}{d\,T}\right)\Delta\,T\right]d\,T = W\,\Delta\,l\,(d\,h),$$

or

$$\left(\frac{d\,Q}{d\,h}\right)\left(\frac{d\,h}{d\,T}\right) = W\,\frac{\Delta\,l}{\Delta\,T}\,\frac{(d\,h)}{d\,T}.$$

In this form $\left(\frac{d\,h}{d\,T}\right)$ and $\frac{(d\,h)}{d\,T}$ are not necessarily identical, for $d\,h$ in the first is the rise in the interval $d\,T$ at the head of the reach and $(d\,h)$ is the mean rise over the whole reach during the same interval. As, however, the corresponding magnitudes of $\Delta\,l$ and $\Delta\,T$ are reduced, the difference between $\left(\frac{d\,h}{d\,T}\right)$ and $\frac{(d\,h)}{d\,T}$ may be made smaller than any assignable quantity and finally disappear from the equation as a differential when $\frac{\Delta\,l}{\Delta\,T}$ becomes $\left(\frac{d\,l}{d\,T}\right)$.

Hence,

$$\left(\frac{d\,Q}{d\,h}\right) = W\left(\frac{d\,l}{d\,T}\right).$$

Now, by its definition here, $\left(\frac{d\,l}{d\,T}\right)$ is the flood movement at that point of the river, or the rate at which equal stages or discharges move down; and, representing its value in feet per second by m, the equation becomes

$$\frac{d\,Q}{d\,h} = m\,W. \qquad \text{II}$$

It may be noted, in the above process, that the parentheses ( ) have been used to indicate definite physical values determined independently. Thus $\left(\frac{d\,Q}{d\,h}\right)$ is given by the discharge curves. On the Lower Mississippi it may vary from 10 000 at low water to 60 000 at high water; or in the Missouri from 3 000 to 40 000; and it is commonly referred to anywhere as the change of discharge per foot of rise or fall there. Again, $\left(\frac{d\,h}{d\,T}\right)$ is the rate of rise or fall at any time. It is in common use as feet per day or inches per hour, but, being expressed here in second units, its value, for a rise of 2 ft. per day, is $\frac{2}{86400}$. And, finally, the $\left(\frac{d\,l}{d\,T}\right)$ is that rate of flood movement which was at

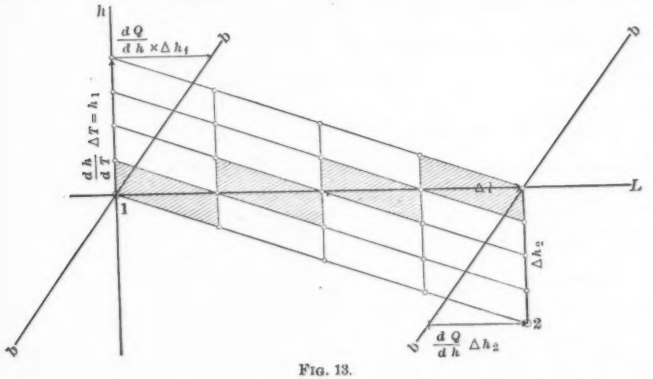

FIG. 13.

last determined with precision, at least for long reaches, in the preceding study of that subject.

With these in mind, the process of deduction is simple enough, and it is seen readily that Equation II is a most general equation of rivers, true for every element of any river from high to low water and from one end of it to the other. But, before going further, it may perhaps be as well to give, in addition, a somewhat more distinctly physical deduction of it.

Taking then, as in Fig. 13, an origin O_1 on the surface at the upper point at a given stage, and the axis of L a length of the river below at the same stage; its level representing the surface slope of the river when the discharges are equal at the two ends, or the river is on a stand

all the way down; then, with the ordinate h measuring the change of stage from this stand, the actual surface of the reach in any given rise is shown by the line $O_1\ O_2$. Taking now $b\ b$, an element of the discharge curve for the reach, its absolute value everywhere at the level $O_1\ L$ being Q_1, and the rate of its change of value $\frac{d\ Q}{d\ h}$, then, if $\Delta\ T$ is the interval which it takes to rise from O_2 to the level $O_1\ L$ and $\Delta\ h_2$ the corresponding change of stage there, at the beginning of this interval the upper discharge is Q_1 and the lower is $Q_1 - \frac{d\ Q}{d\ h}\ \Delta h_2$, the difference being $Q_1 - \left(Q_1 - \frac{d\ Q}{d\ h}\ \Delta\ h_2\right) = \frac{d\ Q}{d\ h}\ \Delta\ h_2$. In the same way, at the end of the interval $\Delta\ T$, the upper discharge is $Q_1 + \frac{d\ Q}{d\ h}\ \Delta\ h_1$ and the lower discharge is Q_1, the difference being

$$\left(Q_1 + \frac{d\ Q}{d\ h}\ \Delta\ h_1\right) - Q_1 = \frac{d\ Q}{d\ h}\ \Delta\ h_1.$$

The mean difference in the discharge between the upper and lower ends of the reach, through the interval $\Delta\ T$, is therefore $\frac{d\ Q}{d\ h}\left(\frac{\Delta\ h_1 + \Delta\ h_2}{2}\right)$, and there is thus brought into the reach during this interval a quantity $\frac{d\ Q}{d\ h}\left(\frac{\Delta\ h_1 + \Delta\ h_2}{2}\right)\Delta\ T$ more water than is carried out. This must fill the volume indicated by the vertical area $\Delta\ l\left(\frac{\Delta\ h_1 + \Delta\ h_2}{2}\right)$ multiplied by the width of the river W.

And equating these two

$$\frac{d\ Q}{d\ h}\left(\frac{\Delta\ h_1 + \Delta\ h_2}{2}\right)\Delta\ T = \Delta\ l\left(\frac{\Delta\ h_1 + \Delta\ h_2}{2}\right) W;$$

or, as before,

$$\frac{d\ Q}{d\ h} = W\ \frac{\Delta\ l}{\Delta\ T} = m\ W. \quad \text{II}$$

The application of this equation may perhaps be best noted by bringing it into comparison with the ordinary hydraulic formula

$$v = c\sqrt{r\ s} \quad (1)$$

$$\frac{d\ Q}{d\ h} = m\ W \quad (2)$$

Thus (1) and (2) have in their order essentially similar physical measures. For instance, v in the river is obtained by measuring the

discharges from high to low water and dividing them by the corresponding areas of their cross-sections; while $\frac{d\,Q}{d\,h}$ is obtained by plotting them to the corresponding gauge readings and differentiating the resulting curve. In the same way, s and m are taken from surface observations; one being a rate of changing level, the other a rate of advancing stage; while r and W are simply characteristic lengths, the first a sort of inverted radius, or area divided by wetted perimeter, the second a plain surface width.

Finally, also, in applying the formula $v = c\sqrt{r\,s}$ to a perfectly general form of flow, where everything may have any arbitrary variation in a given length, subject alone to this condition, even if it expressed everywhere such a variation and the c in it was an absolute constant, it would still be very evident that the element through which it must be taken is not a uniform length element. Thus, if it is to be summed into an equality and averaged, the given length is to be divided into a certain number of elements, each $v\,\Delta\,T$ long, and in each of which it expresses an equality, and the general relation of the whole distance is the average of these elements. That Equation II has exactly the same characteristic may be seen readily by taking it before its reduction, as $\frac{d\,Q}{d\,h}\,\Delta\,T = W\,\Delta\,l$. For $W\,\Delta\,l$ is simply the surface area in the variable elements of length, and $\frac{d\,Q}{d\,h}\,\Delta\,T$ corresponds to the $v\,\Delta\,T$ for each of the constant intervals of time into which the distance may be arbitrarily divided.

Here, however, the parallel ceases, the ordinary hydraulic formula refers in the main to conditions in which the discharge is fixed, such as the discharge of a pipe under a given head, or a conduit with a given form and slope; and it is certainly a most necessary relation there, notwithstanding the fact that its c is a very intricate variable, for Equation II has no application whatever to such a case. But, on the other hand, in rivers Nature offers a condition of an almost continually varying discharge, and, as there is no way by which an irregular volume can be measured more accurately than by taking the amount of water required to fill it, in such a field the fundamental relation between dimensions and the action on them of these varying in-flows and out-flows is not only applicable, but is the only one, in the range which may be covered by it, to be considered.

But, of course, this is not the place to discuss the application of either of these formulas to all the conditions of flow to which they may be applied. And, in fact, in the case of Equation II, there are probably a number of applications which the writer has never yet thought of. He has been interested mainly in applying it to alluvial rivers, and even there, in those reaches in the Lower Mississippi, where the gauge relations are no longer straight lines, it has been already intimated that its application in each case was a special and a single problem. In the whole of this lower river also, with its peculiarly constant flood movement from high to low water, marked with decided differences from reach to reach, there are some questions on which the writer is still working. The only case, therefore, which will be offered here is its application to the general regimen of the Missouri from Kansas City to St. Charles.

The 1879 scales of stage have been given for fifteen locations on this reach, in the former study of discharge and gauge data, and whether they should cover equal time intervals, or simple intervals of distance, is not here a practical question, since their longitudinal sequence has never been determined on this river, and a plain average of them is all that can be obtained, and also, very probably, all that is here necessary.

The average of the equal stages in the tabulation referred to, is 17.1, and the equation of the St. Charles discharge curve in terms of its stage scale is $Q = 0.15\ (h_s + 19)^4$ where Q is the discharge in cubic feet per second and h_s is the St. Charles stage taken from a zero at the low-water level of 5 ft. on that gauge. Calling h_m the average stage of the reach from Kansas City to St. Charles, it is plain that

$$h_s : h_m = 20.0 : 17.1 \text{ or } h_s = \frac{20.0}{17.1} h_m$$

and substituting

$$Q = 0.15 \left(\frac{20.0}{17.1}\ h_m + 19\right)^4$$

from which

$$\frac{d\,Q}{d\,h} = 1.117\ (h_m + 16.3)^3.$$

The corresponding value of flood movement on the Missouri has not yet been given; and indeed there were a number of questions in flood movement which were purposely left to be answered after Equation II had been determined.

Solved for this element, it is $m = \frac{1}{W} \times \frac{d\,Q}{d\,h}$, and with this it only takes a glance at the discharge scales in Fig. 8 to see why the flood movement is so much slower in the Upper Mississippi than in the Missouri. In the same way the marked retardation at the top of the great flood of 1881 on the Missouri is simply the transition from channel filling to valley filling. That the value of W changes sharply in such a transition is plain, while the especially irregular forms of the over-flow discharge curves have been noted. And, in fact, it seems probable that if these forms are ever to be determined satisfactorily it will not be done by measuring their discharges, but by measuring the widths of over-flow and the flood movements, and calculating them.

Again, the 400-miles-a-day flood movement from Baton Rouge to Carrollton is simple enough to anyone who knows the physics of that part of the river. It is well down in the run-out, where the range of high water is being reduced from 50 ft. at Red River to nothing at the Gulf; and the value of $\frac{d\,Q}{d\,h}$ is correspondingly increasing; while at the same time it is also an exceptionally narrow river. It is clear, then, from the equation, that values of m will be found here out of all proportion with its rates elsewhere.

And finally, the constant rate of flood movement from low to high water in the Mississippi will be recognized here as simply a special relation between its general discharge curves and its form. So long as $\frac{d\,Q}{d\,h}$ and W vary in the same proportion m must be a constant, and that they have such a variation vertically in the Mississippi is a fact of its form, and that they do not have such a variation there longitudinally is another; while in the Missouri they have no such special variation together in either direction. But still, all the Missouri scales of stage are linked together in a chain of gauge relations, and there is little doubt that this river has fully as definite a form of its own to correspond with the $\frac{d\,Q}{d\,h}$ given by them, and that its own especial flood movement is there, as certainly as it is in the Mississippi, even though it cannot be determined so accurately in its gauge data.

There is, however, a special variation of m which must be considered. It has been seen, of course, in the deduction of Equation II, that the elements of rise or fall, of whatever magnitude, cancelled out in the final equation, leaving a relation which was immediately

independent of them. This does not mean that the value of m is absolutely independent of the rate of rise or fall, but simply that it is changed by it only as the $\frac{dQ}{dh}$ may be changed by it, assuming, at the same time, that the element of form W at the given level remains the same.

This is a variation of the flood movement from its normal, of the same character as a change of plane in the discharge data, and probably this is the only surface measure in which all those variations of discharge at the given level can ever be followed. It must be looked to also to account for those exceptions to the general flood movement noted in the Lower Mississippi, and the much more frequent changes of the same character in the Missouri. But it should be distinctly recognized simply as a variation from normal flood movement, and it does not coincide with the normal discharge or the normal $\frac{dQ}{dh}$ determined for the river.

It was in this light that the specific determination of the normal flood movement in the Missouri was taken up. The extension-gauge relations from Kansas City to St. Charles had all been plotted for a period of 15 years, showing the character of the whole surface movement there; but it did not seem worth while then to go further in order to pick out the values of its flood movements in all their variations. But when it was understood very clearly that there was a normal movement there which rested on its general form, already so distinctly marked in its discharge scales, the case was different. The extension-gauge relations were then examined and the movements at those periods least affected by tributaries, and best suited to give normal values for different stages, were taken and averaged for the general relation.

This result, expressed in miles per day and taken on the same stage scale in which the $\frac{dQ}{dh}$ of this reach has been given, is

$$m = 70 + 3.27\ h_m.$$

It is thought to be correct to at least within 3 or 4%, but it does not show such limits as distinctly as in the case of the Lower Mississippi. It is the best that could be done with data very imperfectly suited to its determination. Its simple and precise determination in such a river will require other data than those of the ordinary gauge readings.

Equation II, when m is taken in miles per day instead of feet per second, becomes

$$\frac{d\,Q}{d\,h} = \frac{5\,280}{86\,400}\, m\, W,$$

$$\text{or, } W = \frac{16.364}{m}\,\frac{d\,Q}{d\,h},$$

and substituting the values of $\frac{d\,Q}{d\,h}$ and m gives finally, for the equation of the general regimen of the Missouri from Kansas City to St. Charles,

$$W = 16.364\,\frac{1.117\,(h_m + 16.3)^3}{70 + 3.27\,h_m}.$$

This, of course, is simply an equation of dimensions. It gives the value of W from high to low water, and, as such, determines the dimen-

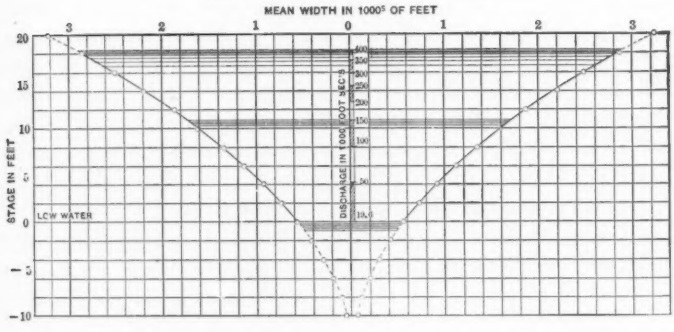

FIG. 14.

sions of the mean hydraulic prism through this reach of river. But no co-ordinates of position have been taken anywhere, and therefore any showing of its mean cross-section must have a further assumption; for any number of areas may be taken between two levels, all having the same value and the same variation, but all differing in their actual forms.

Assuming that W increases symmetrically from an imaginary center line, Fig. 14 shows the mean cross-section of the prism for this reach. Of course, in a divided channel, the actual increase of W with stage, might be altogether toward the center, while at low water especially, its increase would be in any and every direction; but it does not make any difference in what direction it increases, so that the amount and rate of its change are seen distinctly, and this is shown very well in Fig. 14.

Above the water line of the 400 discharge, the river is more or less out of its banks, and that is the limit of this section. What $\frac{dQ}{dh}$ and m may be above that is not known, but it is known that they are not continuous. In the same way there is no guarantee that they would be continuous if they could be observed below low water; but this section is not either of them, but a form of the river, which would be there just the same if all the water on the instant were taken out of it, and this relation has been carried down to 10 ft. below low water, as the most probable continuation of this form as it approached its lower limits.

Between these water lines, however, this regimen of the Missouri is simply measured; not, of course, directly, but as definitely as a distance may be measured with a base and two angles. This measure also has in it probably about the precision of ordinary cross-section data; but it brings out a marked form of bed which all the direct cross-section data has never even suggested. Indeed, it is only in the late surveys on the Lower Mississippi, where the section lines were leveled over all bars and up to the high-water limits, and large means made up from these, that types of this character began to be shown. That the surveys would not give the type accurately is self-evident; for any artificial method of making up the means from them can hardly fail to average values which belong to decidedly different stages. But in the summations of these surveys the essentially triangular character of the Lower Mississippi regimen is well brought out, which its constant flood movements and second-degree discharge curves also give for it.

It is this type of regimen which represents plainly the general equilibrium between the erosive and bar-building forces of the given river; and the type in the Missouri is certainly sufficiently striking. Unlike the plain triangle of the Mississippi, its curvature shows that its forces tend to cut a sharper central thread and build up a widening bar area as the stages are higher. The fact, also noted before, that the low-water level in the Missouri was especially susceptible to the varying forces found in the sequence of high and low-flood years, is here made very plain. Considering the erosion, its action practically consists in cutting down the high banks into this central thread or deeper portion; for the bottom, at the foot of a cutting bank, is rarely above low water. All the caving, therefore, from the cutting banks and bars falls into this central thread and through the low-water season it all stays there,

while the flood period can never fill any of it back above the stage to which the flood reaches.

It is clear that, in the succession of low-flood years, this normal regimen may be changed very materially, this central thread filled up considerably, with all the widths of the lower portion reduced, and the widths of the upper stages increased correspondingly. While again, to maintain the general average regimen of the normal, the action of the succession of high-flood years will be a variation of regimen, from the same causes, but equal and opposite in its character.

Fig. 15, in which the different regimens of 1883 and 1895, before referred to, are given, will serve to show this action. Neither of these regimens has the data needed to determine it absolutely, and they

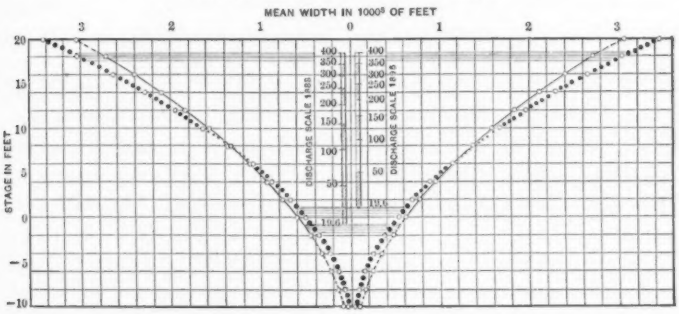
REGIMEN OF THE MISSOURI
1883—— and 1895••••
REACH KANSAS CITY TO ST. CHARLES
MEAN WIDTH IN 1000$ OF FEET
STAGE IN FEET
DISCHARGE SCALE 1883
DISCHARGE SCALE 1895

FIG. 15.

should not perhaps be taken as more than illustrations. Thus they are not definite enough to enable one to say with confidence that the extreme range between erosion and filling over this reach is a variation in mean width of some 640 ft. at high water and some 184 ft. at low water. But they are definite enough to indicate that they are something like this, and that they are certainly complementary and always opposite actions of the same system of forces.

This application of Equation II to the determination of the regimen of the Missouri is simply given as an instance of its uses. It is seen that the whole meaning of the term is changed by it; in the place of regimen being but a convenient expression for a sequence of unknown phenomena with no known relations between them, it becomes the re-

cognized resultant of the counter-actions of erosion and bar building, determined accurately and calculated easily.

Of course nothing is calculated easily from data which are not adapted to it, and the laborious processes here followed should never have to be repeated. Continuous automatic-gauge records, referred to the proper plane, should read the flood movement directly between their traces of stage, and give the values of m with precision from tributary to tributary, and through reaches which have no great changes of flood forms. The longitudinal sequence of the equal stages in these reaches is also easier to get than high and low-water slope lines, and, once determined, its mean value for the reach is probably much more nearly a constant. With these then it is simply a matter of taking groups, each of some six or eight discharge observations, at high-water, at mid-stage and at low-water, to determine positively the scales of stage at one or more locations in a long river. And, transferring these from gauge to gauge, the whole river is then known absolutely, and its form and variations of form can be calculated.

It is this which is offered, in the place of the endless soundings of cross-sections, for the study of the effect of work in the construction reaches, levee effects or reservoir effects, and for all the sequence of changing regimens in alluvial rivers, which perhaps no amount of time and money spent in hydrographic surveys would ever even detect.

It is hardly necessary to say that such data may open new problems in river hydraulics; this method only promises to answer those in sight; and even for those more experiment and more work than is shown herein may possibly be needed. But it is worth understanding clearly that the value of $\frac{dQ}{dh}$ and m for every point of stage is the equivalent of a water-line survey of the reach at that stage. True, the actual measures of distance in the survey have a smaller percentage of error than can be looked for in these calculated results, but measuring up an irregular surface area from the field measures has not. The real defect in the survey, however, is that it can never catch the reach at any one stage, and to reduce it to a uniform stage is difficult, to say the least of it. On the other hand, the flood movement not only takes the areas on planes of equal stage, but, in its sequence from high to low water, readily integrates them into the whole river prism.

Of course, with the regimen given, Equation II may be used to cal-

culate the discharge; but in these changing regimens there is always the corresponding uncertainty in a determination of the absolute value of discharge from gauge readings. No surface levels of any kind in any channel can give its discharge unless some element of the size of the channel is given at the same time, and, in rivers like the Missouri, the first thing is certainly to understand its changes of regimen.

But there are other rivers whose regimens may be fixed, or at least which change so little that it would be enough to check them at long intervals or ignore their changes altogether, and in these, when the regimen of a reach had been once established, the flood movement through it would give a continuous record of its discharge, in all its conditions of rise or fall, probably as accurately as if its waters had been run through some of the best-studied forms of weirs. And if the drainage system of a continent is ever to be taken, and such of its elements determined as can be put together consistently, it would certainly seem that some such method of measuring it up and keeping its record would be absolutely necessary.

DISCUSSION.

Mr. Todd. A. MILLER TODD, Jun. Am. Soc. C. E. (by letter).—The author has contributed a valuable addition to the literature on hydraulics and on the subject of Mississippi River Improvement.

While the principles involved are not by any means new, the method of deduction, and of the transfer of the discharge-gauge relation from gauge to gauge is altogether so, as far as the writer knows.

The results obtained present at once, in convenient form, a discharge-gauge relation for each gauge considered, which is most interesting and useful to river engineers, especially those in charge of levees; for, if the relation is demonstrated to be reliable, it furnishes a clue to possible gauge heights when the river is confined, which, for any very great flood year, has never been done. In other words, say that we know the maximum discharge, due to pass a given place; to what height on the gauge will that discharge raise the surface level of the water?

The question of possible gauge height is a paramount one to the levee engineer when it becomes necessary to establish grades for his levees, when the river is confined throughout its length, or any portion of its length.

While the paper may not give conclusive evidence on the subject, yet we may apply the discharge-gauge relations, and compare results with those obtained by different methods of computation; which methods, in every case, are crude and more or less hypothetical. We may thus check former calculations, or inaugurate further investigations and calculations, in order to reconcile the discrepancy if any is found to exist.

It is very true that it is difficult and expensive to measure the discharge at a sufficient number of points, and to study very closely the progress and attendant phenomena and laws of water in its passage through great channels like the Mississippi below Cairo. Observations which have been made seem discordant and discrepant, and methods have been crude, but, beyond a doubt, work which has been done along this line cannot be praised too much. From the very first discharge measurements taken in the Mississippi, the methods have been improved upon, and the consequent results enhanced in value, up to the present time; and the writer thinks this improvement, both in methods and results, will continue. Therefore, he would not too hastily declare the author's method of studying river phenomena one which could supersede the old altogether; but, as before stated, the author's principles, properly applied, should furnish a valuable check and help to river engineers in future studies and investigations.

Mr. Todd.

The author gives what he calls (average) "discharge scales." The writer has at hand a "Monograph" by the author "on Reservoirs and their Effects on the Floods of the Mississippi System" in which these "scales" * are plotted to a much larger scale, and they are very interesting to study. The writer has tested them with observed discharges taken at various times, and has found that the scales agree very closely with the actual results obtained from field discharge observations, and, in all instances and at all stages, they seem to show the average discharge.

In a number of lines of study, these average scales, as they are, should give correct results. But it occurs to the writer that, in many instances, before the levee engineer can accept the plotted values as correct, he must take into account the local condition of the water surface in the reach he has under consideration; whether it is rising, stationary or falling, and the rate of rise or fall; these are conditions which affect the flow. The author calls attention to the fact that, "the same change in the conditions of flow which would change the surface level by 1 ft. in a river from 50 to 100 ft. deep, would only change it by 0.1 ft. in a river from 5 to 10 ft. deep." So that, while the difference in discharge, due to whether the reach is rising or falling, is small at low stages, it becomes, at high stages, of such magnitude that it cannot be neglected. The author states that the time occupied by a change of level, passing through a given reach, is absolutely constant; but the writer understands that most, if not all, river engineers hold, from observation, that a fall travels more slowly than a rise; and, even at low stages, a rise in the reach, if the latter is long enough, will overtake a fall. Thus m in the author's formula cannot strictly be reckoned as a constant.

The author calls attention to a change of plane at Arkansas City, and offers possible explanations as to the cause, neglecting entirely what seems to be the true cause of the change; at any rate, as eminent an engineer as William Starling, M. Am. Soc. C. E., who is a recognized authority on river subjects, accounts for the identical change, and plots the identical figure used by the author, Fig. 7. In a paper by Major Starling,† Fig. 24, on page 450, shows, evidently, that the change of plane is that which recurs regularly, shifting from one to the other, according to whether the river is rising, stationary or falling.

In that valuable paper Major Starling undertakes to foretell the probable gauge height of maximum flood discharge in reaches where the floods have never been confined. He frequently calls attention to, and takes into account, the difference in discharge, at equal stages,

* House Document, No. 141; 2d Session, 55th Congress, Plates 8 and 9.

† "The Discharge of the Mississippi River," *Transactions*, Am. Soc. C. E., Vol. xxxiv, p. 347.

Mr. Todd. due to "rising river" and "river after rise," and he always obtains two curves, calling them the "two branches."*

In Fig. 16 the writer has plotted the discharge observations referred to by both the author and Major Starling, and has fitted and marked distinctively, curves through certain periods of rise and fall.

It will be seen that the fall on February 4th had something to do with the shifting of the discharge-gauge relation up to the stage on that date, and there can be plotted, through the points indicating a fall, a distinct and separate curve from that drawn through the points marking the rising stages. This second branch is traced down as long as the river falls. A rise commences on March 4th, and, instead of the discharge-gauge relation agreeing with the rise of December 30th to January 18th, the discharge seems to pass at considerably higher gauge readings. Under the circumstances, the writer thinks the foregoing fact should be expected. The average datum area was practically the same during both rises; while, from December 30th to January 18th, 19 days, the river rose 25 ft., or an average of 1.3 ft. per day, and from March 6th to March 23d, 17 days, it rose only 13 ft., or an average of 0.7 ft. per day.

For any portion of the reach in which Arkansas City is situated, if there are gauge relations, which cannot be doubted, then, for a rise of 1.3 ft. per day, the slope is bound to be greater than for a rise of only 0.7 ft. per day. Assuming, as the author does, that the miles per day traveled by a change of gauge height in a given reach A B, Fig. 17, is constant; assuming, also, that 1 ft. rise at A is equal to the same amount of rise at B, or that the gauge relation of B to A is 1, and that the time interval is 1 day; also, suppose that, on a given day, the gauge at A shows a rise of 0.7 ft., then the full line A_1-B will represent the general slope of the river on that day; the next day there will be 0.7 ft. rise at B and also at A, and for all succeeding days, as long as the increment of 0.7 ft. daily at A obtains, the slope on those days will continue parallel to A_1-B. If a 1.3-ft. rise is recorded at A, the dashed line A_2-B will represent the general slope for that day, and for all succeeding days of the 1.3-ft. rise. Now, for example, assume the distance, A to B, equal to 60 miles, then the increase of slope of the 1.3-ft. rise over the 0.7-ft. rise is 0.006 ft. per mile. The slope being a function of discharge, the discharge-gauge relation varies accordingly.

It is the writer's opinion, that, as long as the river is rising or falling at nearly a constant rate, if true discharge measurements could be obtained and plotted, the points would lie in two perfectly regular curves, one representing the rising river, and the other the falling river, for that rate of rise or fall; and points for any other rate of rise or fall will depart from these curves more or less. So that, for

* "The Discharge of the Mississippi River," *Transactions*. Am. Soc. C. E., Vol. xxxiv, p. 465, Fig. 29.

the conditions of the water surface, with constantly varying rates of rise and fall, the actual curve of discharge-gauge relations is never a regular one. But, between an imaginary curve, representing the discharges passed throughout a rise to extreme flood height, rising at the constant rate of the greatest known average rise per day, and a curve representing a fall under the same conditions, there may be Mr. Todd.

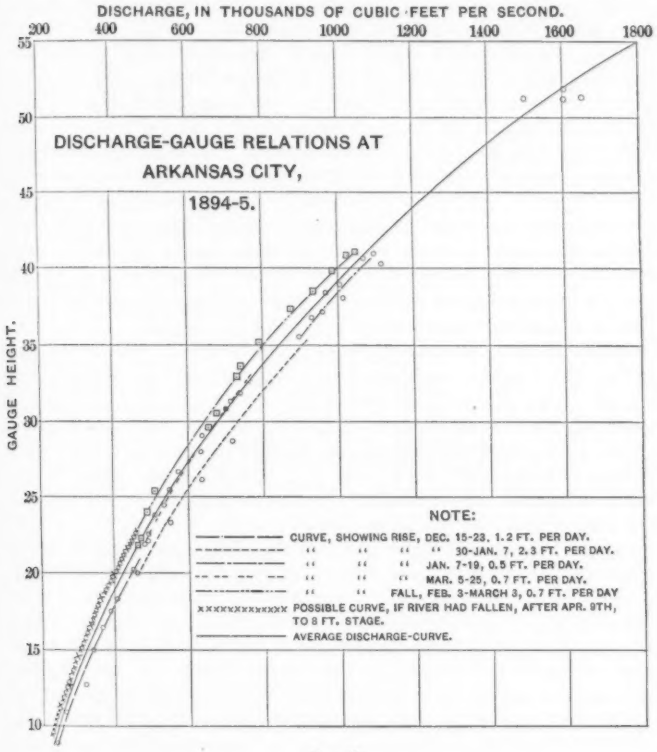

FIG. 16.

constructed a mean curve which would give a basis by which to reckon discharge-gauge relations under all varying conditions of flow.

If the observations from which Fig. 16 was derived had been continued, and the river had fallen to a stage of 8 ft., without being influenced by any sudden rise or fall, the points obtained, the writer thinks, would lie about where the x marks are indicated. Taking these points into consideration, and also the points plotted at 50 to

Mr. Todd. 51.5 ft., obtained from discharge observations of the Mississippi River Commission in 1897 and 1898,* and projecting a mean curve through all the points obtained, as indicated by the full line, this curve, in similar form, would correspond to the author's discharge scales, and is about what he would obtain for Arkansas City, if not exactly the same.

Fig. 16 shows plainly the departure of some of the observed discharges from the average discharge-curve. For investigations requiring the summation of discharge over a period covering a rise and the corresponding fall to about the same stage, the scales constructed by the author cannot be improved upon. But, as before stated, they will not do where the discharge covering any given day, or the maximum discharge, is wanted. To render the average discharge curve useful in such instances, corrections must be applied to all stations, at least below Cairo, in the Lower Mississippi; and, the writer thinks, that the correction depends principally on the slope of the surface of the water in the reach in which the gauge station under consideration is located, and at the time the discharge corresponding to a given gauge height is desired. These slopes can be had roughly, at the present time, and, if the gauge stations were located and kept, as suggested by the author, the slope between them could be ascertained accurately at all times.

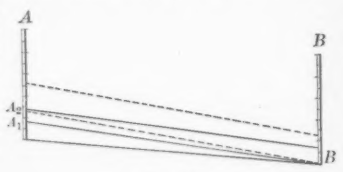

FIG. 17.

The author states that in certain instances corrections should be applied, and undoubtedly he has taken all the foregoing facts into consideration, and probably has a method of applying the correction, not hinted at by him or conceived of by the writer. Possibly the writer, in his limited study of the problem, has laid too much stress on the slope as due to the many various conditions of the river, owing to the rise and fall alone; at any rate, he hopes that the author will state, in his closure, what his corrections are, and to what extent they will affect the quantities given by the discharge-scales.

There is one thing certain, as the author states, in transferring the investigations down the river below Cairo, the problem becomes complex indeed, due, principally, to the enormous number of variations and combinations of slope and momentum possible, according to the condition of the water level, not only in the single reach, but probably in several reaches above, which may affect the discharge materially; and also to the conditions and stage existing in the various tributaries. However, the only way to arrive at any tangible results, in clearing up and solving this great problem, is to keep hammering

* See corresponding reports of the Mississippi River Commission.

away at it; and the more we hammer, the sooner the desired end will be attained. After the levees have been maintained throughout several extreme high waters, we may expect to have some light on the subject. Heretofore, all the extreme high waters, after passing Cairo, have first been restrained in one place, and then have broken the levees and inundated basins in other places, the conditions of no one year repeating themselves the next; and it is impossible to study, with any degree of satisfaction, a flood spread over 100 000 square miles and upward. One flood, that of 1898, of comparatively short duration, which came within 2 ft. of the 1897 water at Cairo, was restrained successfully where leveed. The St. Francis levees were not completed, within some 100 miles of Helena, but the remaining portion is now being constructed. This basin being for a great part still open, the question of the effect of its closure on the flood plane, or gauge at Helena, is still problematical. Mr. Todd.

So that, for many years to come, the subject presented by the author will be an important study to all engineers interested in the Mississippi River problem, and it is to be hoped that he will give us, at some future date, the results of his further investigations.

GEORGE W. RAFTER, M. Am. Soc. C. E. (by letter).—This paper is an admirable illustration of the present tendency in hydraulics, not to attempt to express complex relations drawn from a large number of cases by a single formula, but rather to work out each case by itself, on its merits. The recognition that the discharge of a stream for a given gauge height will be different for rising from what it is for falling stage, with each in some proportion to rise and fall, is a case in point, as is also the discussion of the effect of "bed in train." Frequently, the tendency has been either to include all such phenomena in a single expression, or to ignore them entirely. The best illustration, however, is found in the conclusion of the paper, that there is a particular equation which expresses the hydraulic relations of rivers better than the formulas in common use, but which does not in any degree apply to pipes, conduits and uniform reaches of straight channel. Mr. Rafter.

While the paper thus illustrates a desirable improvement in hydraulic studies, it contains, further, a series of generalizations which assist one materially in comprehending the complex series of physical facts entering into the flow of a large stream, where bends, irregular bed and other disturbing influences tend to complicate the phenomena. For all such, notwithstanding current practice, it is well to recognize that $v = c\sqrt{r\,s}$ can have at best only casual application, and as a demonstration of this point Mr. Seddon's paper can hardly be excelled. But for straight reaches of artifical channel with uniform cross-section, the conditions of flow are so different from those of meandering and silt-bearing streams, that deductions applicable to one may not apply

Mr. Rafter. in any degree to the other. For such a channel, the theory of velocity-slope relations becomes, the same as for pipes and conduits, all important. As shown by the author, the views expressed in the paper do not apply to these cases, but are to be considered as confined to large streams with relatively flat slopes. Nor, so far as the writer can now see, will they apply to small streams and mountain torrents, for both of which the Chezy formula is more nearly applicable. This, however, is merely in line with the proposition to, so far as possible, work out formulas suited to each specific case.

Mr. Le Conte. L. J. Le Conte, M. Am. Soc. C. E. (by letter).—It has been generally known that there exists a large amount of valuable information on the files of the several district offices in the Mississippi Valley which, from lack of funds, has never been properly studied and digested by any competent person. The author has undertaken this task, and has presented many facts and important conclusions.

He brings out strongly the important feature that local slope is of no importance in the formulas for stream flow, because it is an unstable as well as a secondary result of local changes in cross-section; and since the cross-sections themselves are constantly changing, the local slope also changes correspondingly, and yet the discharge remains constant all along the entire reach. These potent facts show how careful an engineer should be in selecting the upper and lower limits of the characteristic reach, in order to ascertain, for the entire reach, the true average fall, the true mean characteristic depth, and the corresponding constant discharge. That is to say, he must not allow himself to be misled by any non-characteristic local slope or cross-section, their lack of stability being in itself sufficient evidence of their secondary character. As an extreme case, showing the lack of influence of surface-slope, we have at Donaldsonville an actual reverse-slope extending up the river for 20 miles or more, and yet the great quantity of water moves down the river with the same discharge as where the slope is down stream.

The author's system of triple gauges at each station is certainly the most rational device, and, at the same time, gives the best and quickest results. The amount of useful information which can be obtained by means of such simple apparatus is remarkable.

The author's comments on levee effects, in always lowering the low-water plane and thus facilitating flood-water propagations down the river, will be welcome news to river engineers as well as riparian land owners. It is a matter of paramount importance in reclamation works, and should command the closest attention. The whole story is shown in an indisputable form by a study of the discharge scales for the past ten years.

The author's statement of the facts pertaining to flood movement and his mathematical deductions therefrom are certainly interesting

and valuable. The full value of their bearing upon the actual flattening out of the flood waves and the correspondingly rapid flood-propagation down the river channel are matters of great importance when dealing with large rivers, and the writer thinks that, at least in the lower divisions, even of rivers of ordinary size, the influence of this flood-pulse has been largely underestimated. The author emphasizes the fact that in the Lower Mississippi the prodigious flood-movement cannot be explained by the direct flow of the waters down stream, because the flood-wave crest travels down the river four times as fast as the water. This feature, together with the mathematical deductions, shows unmistakably that a close relation exists between these flood-propagation phenomena and the laws governing tidal propagation in shallow estuaries and tidal rivers; the only difference being that the flood-pulse originates in the upper river and is propagated down stream, while the ordinary tidal-pulse originates in the ocean and is propagated up stream against the flow of the current. Mr. Le Conte.

It is to be hoped that the author will continue to give us information in this line and thus enable us to weed out the weak points in existing standard formulas based on experiments of small scale.

JAMES A. SEDDON, M. Am. Soc. C. E. (by letter).—From the limited discussion offered, the writer has come to question whether he really has succeeded in making the hydraulic system, presented in his paper, as clear as he thought he had. As the field of river data has in it certainly a good deal of brush wood, in which even the trained investigator may lose himself utterly for a time, and which probably deters many from ever entering it, the writer has concluded that he can best serve the purposes of a discussion by presenting briefly the outline of his system in its application to a simpler case, and one with which engineers are more generally familiar. Mr. Seddon.

Assuming for this a flume, say 5 ft. wide, set on some uniform grade, which it is not now necessary to consider; and taking 10 000 ft. of it as the reach through which observations are to be made; automatic gauges with zeros at the bottom of the flume set at the upper and lower end of it; the water run out and the whole thing ready for the experiment.

The flow then would be started, very little at first, increasing steadily until the flume was running full for a time, and then gradually shut off and the water again drained out of it. During this time the records of the two gauges would be the only data required to determine the discharge of the flume at any and every level and through all these variations of slope found in its filling and emptying.

For instance, each automatic gauge would trace the changing level to a common time scale, and should, of course, be set accurately to give this from the bottom of the flume, enlarged if necessary, and so proportioned to the time scale as to mark the coincident time and level

Mr. Seddon. most distinctly. Then, say, from the trace of the rising level at each gauge, the data of Columns (1), (2) and (3) in Table No. 2 are taken.

TABLE No. 2.

Water level in flume h, in feet.	Time at which Level is Reached on Gauges.		Difference of time, in seconds. $= \Delta T$.	$\frac{10\,000}{\Delta T} = m$.	$m \times 5 = m W$.	Discharge, in cubic feet per second $= Q$.
	Upper gauge.	Lower gauge.				
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0.1	6 00 00 A.M.	8 34 20 A.M.	9 260	1.08	5.4	0.5
0.3	6 40 00 "	8 54 30 "	8 070	1.24	6.2	1.7
0.5	7 20 00 "	9 19 00 "	7 140	1.40	7.0	3.0
1.0	8 10 00 "	9 42 40 "	5 560	1.80	9.0	7.0
1.5	9 00 00 "	10 15 50 "	4 550	2.20	11.0	12.0
2.0	10 00 00 "	11 04 10 "	3 850	2.60	13.0	18.0
3.0	12 00 00 M.	12 49 00 P.M.	2 940	3.40	17.0	33.0
4.0	2 00 00 P.M.	2 39 40 "	2 380	4.20	21.0	52.0
5.0	4 00 00 "	4 33 20 "	2 000	5.00	25.0	75.0

From this, Columns (4), (5) and (6) are readily computed; and, platting these values of $m\ W$ to h gives the line shown on Fig. 18, which is expressed by the empiric equation $m\ W = 5 + 4\ h$.

Then from Equation II,

$$\frac{d\,Q}{d\,h} = m\ W = 5 + 4\ h,$$

and integrating,

$$Q = 5\ h + 2\ h^2 \pm c.$$

In the more general case, where h is the stage scale and is taken from a zero at an arbitrary low water, c, the constant of integration, is simply the regular discharge of the river at that level, upon which the variations of discharge given by the flood movement are imposed. But here, where h is taken from the bottom of the flume, and the values of Q and h become zero together, c is then zero, and the complete discharge equation is $Q = 5\ h + 2\ h^2$, from which the values of Q given in Column (7) are computed, and platted on Fig. 18 show the discharge curve as determined.

This, of course, is merely an illustration, and may not correspond with any actual case; and, indeed, no case is actually fixed until the grade of the flume is given, which has not yet been even considered. It simply presents the process of determining, in a channel of given dimensions, a whole discharge sequence at once merely with the records of two gauges.

Neither does the writer claim that it is a better measure of the discharge here than the ordinary method of observing the velocity at a number of points in the section and from these computing the flow; or the alternative of running a steady flow into a basin of a given size,

and from the filling in the period calculating the discharge. Considering the wide range of discharge values which the proposed method covers in a single operation, with no meters to rate or observers' errors to question, and the whole thing reduced with little more work than that of a single discharge observation, if the writer had any experiments on the flow of conduits or flumes to make, he would certainly try it. But, until it is tried, he is not ready to consider its precision. All this rests upon the precision with which the time element involved in m may be determined, and is a matter yet to be tested. Mr. Seddon.

But this is not an untried field in the writer's studies of rivers. He has shown that here in favorable cases m may be determined within limits hardly exceeding 1%, and even on the Missouri probably close to 3 per cent. This has been done also simply with the ordinary gauge data, and how unsuitable this is for such determinations should be understood. In the first place, the gauge observers are local men employed in other occupations and paid a small sum monthly to take the readings and send them to the different offices, and there is no guarantee that they do not generally read their gauges earlier or later than the set time, by an hour or so, as suits their convenience. Again, the water surface is frequently quite rough, and even the most careful man may make a mistake of a tenth or more in estimating its level, with waves running a foot high. And, finally, the gauges themselves are often set with no nice regard for any real precision in their readings, taking in the pulse of an eddy, or the extreme variations of slope through a bridge span, and in the case of cable gauges, all the temperature errors incident to measuring down from the lower cord of a bridge to the water surface with a weight at the end of a wire.

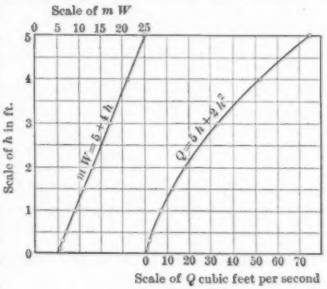

FIG. 18.

The precision attained in the determination of m, with such data, leaves no doubt whatever that surface observations, planned and taken with that specially in view, would bring this proposed measure of discharge into a field of more reliable and exact data than the best that has yet been taken, not to speak of the whole mass collected, which now requires years of study to sift in order to form some idea of what is reliable in it.

This, however, is simply the case where the dimensions of the channel are given. In tidal rivers, for the determination of their peculiar form of discharge curves, with a zero at both the upper and

Mr. Seddon. lower levels joined by plus and minus branches, probably this process would be all that was wanted. But in other rivers it may be desirable to first calculate dimensions; and, going back to the flume to consider such a case, it is plain that the discharge curve of Fig. 18 is not the curve for the head of the reach nor the foot of it, but for some intermediate average; and that this again changes with the grade of the flume and different rates of filling and emptying.

The only constant in the matter, with a set grade, is the intermediate value of discharge between equal rates of rise and fall, or the uniform flow at the different levels, and this corresponds to the mean of the rising and falling flood movements. In the case of the flume, very possibly, accurate dimensions might be determined with simply rising discharges observed in the middle of the reach and the coincident flood movements through it; but in the river, where a general stage scale for the reach is also to be gotten, the process of standard curves and mean m^s is preferable.

However, the dimensions once determined in this way, if the form of the river is fixed, W at any time, in the equation $\frac{d\,Q}{d\,h} = m\,W$, is given, and simply measuring m then will settle all questions of what effects the larger slope on the rise and the less on the fall may have in that reach. So far as his own studies go, with these rivers of fixed regimen, the writer would say that one reach may show these effects and the next may not; and that this depends on whether a general slope controls the flow there or special sections. But it is certainly a field that needs further observations.

In a general way, also, the same may be said of alluvial rivers. But since the writer in 1885 turned a given flow down an inclined plane covered with river sand, and found that it came to about the same velocity no matter what inclination he gave it, he has given up trying to explain all the phenomena of alluvial river flow with velocity-slope relations. And, indeed, he thinks it is well here for every investigator to first try and see broadly what slope does not explain before he puts any great amount of time into fitting local data thereto.

Take a case from the Arkansas City, 1884–85, discharges illustrating it with Fig. 19, in which scales are ignored for the sake of bringing the matter out clearly. It is seen that this change of plane, or different levels of the same discharge in the first and second periods, comes in between Helena and Arkansas City and from there on is continuous to Vicksburg. If the different slopes on the rise and fall are taken to explain it from Arkansas City down, it must at the same time be explained how this flood with about the same slope differences shows no such marked change in the discharge levels from Cairo to Helena. And, further, in the same connection, while the writer does not draw conclusions from the data of single sections, it may yet be noted,

that Mr. Starling's coefficients* for this Arkansas City section, with these slope differences taken into account, show an even more marked difference between the two periods than that given by the discharges. Mr. Seddon.

The writer calls this a change of plane, and, following the flood from Cairo down, as he finds no general difference of any such magnitude that he can assign to the varying slopes of the rising and falling river, he does not recognize them as the cause of it. He does not mean that such effects are not there, but simply that they are not in this proportion. But these are not the only changes of slope involved in the matter. In Fig. 19 a change in the slope of the discharge levels from Helena is shown, and it has been stated that this corresponds roughly with a flood out of the Arkansas. It should also be noted that in such a period when the discharge is 600 at Helena it may be 700 to 800 at Arkansas City; and this change in the discharge level is then but a fraction of the actual change in the surface slopes leading down to it. Had Mr. Starling, in place of taking his slopes from Arkansas City to Greenville, taken them from above to the mouth of White River, where this change really comes in, he would have found a much better ground on which to base these discharge differences.

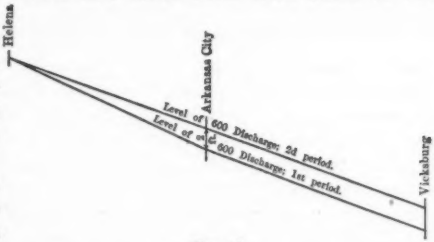

FIG. 19.

But why, when such a change has once come into the river, it seems to hold for a considerable time after the tributary flood has run out, and is continuous, apparently without change in its magnitude, for hundreds of miles down, are matters which the slopes do not explain. However, to answer Mr. Todd's question as to what the writer's system of corrections is: He will have to call them, for the present, simply accumulated experience of where and under what conditions such changes are likely to come in, and their probable magnitudes. He does not feel that he has quite gotten to the bottom of this phenomenon, and is not ready to say that any single cause, that is altogether satisfactory, can be assigned to it.

Going back, however, to the flume for comparison, it may be seen that while the general grade in the alluvial river is also fixed; it is still the case where at any location the bottom, or the zero of discharge, from time to time may be shifted. Simply shifting the discharge curve as a whole up or down, the writer has found covers at least the

* "The Discharge of the Mississippi River," *Transactions*, Am. Soc. C. E., Vol. xxxiv, Fig. 20.

Mr. Seddon. greater part of the phenomena; and as this is done in a minute, and records definitely the change in both its extension of time and length of river, it suits his method of first collecting facts on which to build his theories. It, of course, makes the problems more complicated, but he has indicated the general methods of meeting them, and that is as much as can be expected in a new field such as this is. That where such changes occur, the slope is altered, is evident, but the writer takes these changes of slope as effects; the field in which slope is a primary cause he takes to be quite a different matter.

In that lies the explanation of all the general forms or types shown in the different regimens of different rivers, and the writer does not come to it in the paper presented; he has carried the subject no further than the case of the flume, where the grade was not even considered. He offers, however, a system in which these regimens may be determined, and, indeed, which promises to bring rivers into a much more exact field of calculation than that found in any other line of hydraulics; certainly the more precise determination of m is a subject for further experiment, and the application of the equation $\frac{d\,Q}{d\,h} = m\,W$ must be varied to suit the cases. But taking the given regimen of the Missouri River from Kansas City to St. Charles as an example, perhaps something of the dynamics, in which this, as a whole, stands as an equilibrium, may best illustrate his view of it.

First, the energy of the discharge and fall in this reach varies from about 750 000 H.-P. at low water to about 15 000 000 H.-P. at high water. This, then, tears down on an average about 120 000 000 cu. yds. of bank annually, or an amount which, dumped in year after year, is enough to fill this given prism up solid to some 12 ft. above low water in the twenty years in which the Government has been working on it. That it has not changed it perceptibly, makes the fact very plain that it has simply been built back again on banks and bars in other places.

Of course, a considerable part of the observed sediment in the water lies simply in this action, and for the reach from Kansas City to St. Charles, as a whole, may be said neither to come into it nor to go out of it. But taking this in about the proportion of the sediment that will settle from the water with the least check to its velocity, and which can hardly have come from distant head waters, leaves the true sediment for the year about 130 000 000 cu. yds. carried in suspension. And certainly the percentage of this which may be dropped in the reach is an additional load on the bar-building forces.

But a good part of this matter carried in suspension will not settle in a reservoir after standing 24 hours, while, even as a whole, it is but little more than the erosion on this 333 miles of the river, and, for the total alluvial stretch of the Missouri, is a minor fraction in comparison with it. Indeed, the fact of the almost uniform grade on which this

river is set leaves little doubt that it has been long ago leveled up to bring the sediment entering it into practically a thorough transportation; but, in any event, in all such cases the probable proportions are such, that if no changes from erosion are shown in the 10 or 20-year periods, about 1 000 years is the least time in which material effects may be looked for from the matter carried in suspension. Mr. Seddon.

Of course. all this only shows when the long reach of river is taken into view altogether. At a given location the erosion may be small, and the bar building large, while the sediment carried through in suspension is so great in proportion to both of them that anything seems to be possible from it. It is not until the engineer gets away from actually looking at it, and sees, in his mind, the hundred miles of river as a whole, as he sees the orbit of the planet, that he begins to find himself in the presence of the real dynamics of rivers, and can fairly value the processes given, in which these equilibriums may be measured, and their variations traced from river to river, and from season to season; a matter that at first looks as hopeless to him as chaining the distance to the moon, but which, after all, is very easily measured when we know how.